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Abstract

In this study, we investigate the strategic manipulation of two-stage voting procedures with the nomination process. In the model, some alternatives are first nominated based on voters' opinions. Subsequently, the voting outcome is chosen from the set of nominated alternatives, based on voters' preferences. The model has a richer structure than that of the "preference-approval" model. We apply the notion of strategy-proofness to the model, and define a weakened version, called opinion-based strategy-proofness. In addition, we propose a new notion of non-manipulability for strategic nominations, which we call stability. We show that a Gibbard–Satterthwaite-type impossibility is still valid in the proposed model, but identify an efficient anonymous two-stage voting procedure that is opinion-based strategy-proof and stable.

JEL classifications: D71, D72

Keywords: strategic nomination, opinion-based strategy-proofness, stability, two-stage voting procedure, strategy-proofness

1 Introduction

Every non-dictatorial voting procedure can be manipulated by a voter who misrepresents his or her preference in order to create a more favorable voting outcome. This negative result is formally shown by Gibbard (1973) and Satterthwaite (1975). It is widely recognized that the Gibbard–Satterthwaite theorem is considerably robust in the sense that the impossibility is not resolved in spite of various attempts.[1]

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[1] See Barberà (2010) for a survey of the literature on the Gibbard–Satterthwaite theorem and strategy-proof social choice.

However, the aforementioned manipulation of voting procedures is only one strategic aspect of collective choice, which usually incorporates the voting process (i.e., choosing an alternative from a given list of candidates) and the nomination process (i.e., identifying a list of candidates). Strategy-proofness is central to the non-manipulability of the voting process; the manipulation of the nomination process is treated in a different context, called "strategic nomination," and has been analyzed independently of strategy-proofness.[2]

This study extends the notion of strategy-proofness to a broader voting procedure that includes the nomination process, and investigates whether strategic manipulation of the extended voting procedure is possible. We suppose that collective choice occurs through a two-stage voting procedure. In the first stage (nomination process), some alternatives are nominated by aggregating the voters' opinions. These opinions are positive or negative views about which alternatives are eligible as candidates for collective choice. In the second stage (voting process), a single alternative is selected from the set of nominated alternatives by aggregating the voters' preferences.[3]

Our model has a richer structure than that of the preference-approval model developed by Brams and Sanver (2006, 2009). In the latter model, voters simultaneously express their preferences that order the alternatives and their evaluations that approve them as acceptable or unacceptable. Here, evaluations are technically equivalent to opinions in the proposed model. Note that the preference-approval model does not include a nomination process, because collective choice occurs through a one-stage voting procedure in which an alternative is selected from a fixed set of alternatives.

Our notion of strategy-proofness requires that two-stage voting procedures be non-manipulable. This means that voting outcomes do not become more favorable in terms of a voter's preference when he or she misrepresents his or her opinion, preference, or both. Therefore, strategy-proofness in the proposed model is a non-manipulability notion in terms of both the nomination process and the voting process of the two-stage voting procedure. If the set of nominated alternatives is given after voters express their opinions, then strategy-proofness in the model is substantially equivalent to that of the standard Arrovian social choice model.

An advantage of this study is that we introduce a non-manipulability notion that is weaker than strategy-proofness in the model: Suppose a voter attempts to manipulate the voting outcome by misrepresenting his or her opinion, preference, or both. The voting outcome does not always change from a negative to a positive alternative in terms of his or her opinion, even when it becomes more favorable in terms of his or her preference.[4] Therefore, if this does occur, then

[2]See Campbell (1979), Dutta et al. (2001, 2004), and Dutta and Pattanaik (1978) for studies related to strategic nomination.

[3]This two-stage voting procedure is a reduced form of the collective choice framework developed by Iwata (2016, 2018), which Iwata (2016) claims is reasonable from both descriptive and normative viewpoints.

[4]We assume that each voter always prefers positive to negative alternatives in terms of his or her opinion.

the gain from the manipulation will be sufficiently large. Thus, he or she has a strong incentive to manipulate the outcome. *Opinion-based strategy-proofness* requires that two-stage voting procedures be non-manipulable only in this strong sense. That is, the voting outcome does not change from a negative to a positive alternative in terms of the voter's opinion when he or she misrepresents his or her opinion, preference, or both.

In addition, we explicitly describe agenda formation via the nomination process. This broadens the possibility of agenda manipulation via strategic nomination. We propose a new non-manipulability notion in terms of the nomination process. Suppose that a voter influences the voting outcome by manipulating the list of candidates. That is, he or she attempts to change the list (i.e., add or remove candidates) by changing his or her opinions.[5] *Stability* requires that the voting outcome remains the same if it is still nominated when some voter changes his or her opinions, under fixed voters' preferences. Therefore, stable two-stage voting procedures are immune to agenda manipulation via strategic nomination.

Unfortunately, we show that it is difficult for two-stage voting procedures to satisfy strategy-proofness in the proposed model. Theorem 1 states that the joint satisfaction of strategy-proofness and stability implies that voting outcomes must be independent of the opinions of voters. We call this property *opinion invariance*.[6] However, it is difficult to derive a positive implication for opinion invariance in the proposed model, because it follows from opinion invariance that the voting outcomes must always be chosen from the intersection of any two sets of nominated alternatives, which ought to vary with the opinions of voters. Therefore, if the intersection of two sets is empty, then the two-stage voting procedure trivially violates opinion invariance. Thus, opinion invariance is not a plausible property in the model. This implies that we have to exclude either strategy-proofness or stability from the list of axioms in the proposed model.

Theorem 2 shows that there exists a strategy-proof two-stage voting procedure that is efficient and non-dictatorial if and only if two alternatives are nominated when every voter has a positive opinion about each alternative. As a corollary to Theorem 2, it is possible to show that if at least three alternatives are nominated when every voter has a positive opinion about each alternative, then every efficient and strategy-proof two-stage voting procedure is dictatorial. Thus, we show that a Gibbard–Satterthwaite-type impossibility is still valid in the model. In fact, this result has a stronger implication than the Gibbard–Satterthwaite theorem in the standard Arrovian social choice model, because it shows that a dictator exists in the voting process, and that the dictator controls the nomination process.

[5]Another type of agenda manipulation seeks to change the order of successive pairwise comparisons of alternatives; see Apesteguia et al. (2014) and Barberà and Gerber (2017).

[6]Opinion invariance is technically equivalent to the approval invariance property in the preference-approval model, proposed by Erdamar et al. (2017). Approval invariance plays an important role in reducing the preference-approval model to the standard Arrovian social choice model; however, opinion invariance does not play such a role.

Although Theorems 1 and 2 have negative implications for strategy-proof two-stage voting procedures, we obtain a possibility result if strategy-proofness is weakened to opinion-based strategy-proofness. Theorem 3 shows that there exists an efficient and anonymous two-stage voting procedure that is opinion-based strategy-proof and stable.

Theorem 3 has an opposite implication to the impossibility result in the preference-approval model. By considering the opinions of voters in the proposed model as the evaluations of voters in the preference-approval model, we can formalize the notion of non-manipulability in terms of the (one-stage) voting procedures in the preference-approval model, which correspond to strategy-proofness and opinion-based strategy-proofness, respectively, in the proposed model. In fact, Erdamar et al. (2017) refer to these as strategy-proofness and evaluationwise strategy-proofness, respectively, in the preference-approval model. They show that every efficient and anonymous (one-stage) voting procedure is incompatible with evaluationwise strategy-proofness if a preference domain is sufficiently rich in the preference-approval model.[7] Thus, by introducing a nomination process, we show that their impossibility in the preference-approval model does not hold in the proposed model.

The remainder of the paper is structured as follows. Section 2 introduces the basic model. Section 3 discusses related literature. Section 4 presents the proposed axioms defined for two-stage voting procedures. Section 5 provides our main results in this study, and Section 6 concludes the paper.

2 The model

Let $X = \{x_1, x_2, \ldots, x_m\}$, with $\#X = m \geq 3$, be a finite set of potentially feasible alternatives, where # denotes the cardinality of a set. If the numbering of alternatives is not needed, we may replace $X = \{x_1, x_2, \ldots, x_m\}$ with $X = \{x, y, \ldots, z\}$. Let A denote a non-empty subset of X. Let $\mathscr{X}$ be the set of all non-empty subsets of X. Let $N = \{1, 2, \ldots, n\}$, with $n \geq 2$, be a finite set of voters.

We introduce a two-stage voting procedure: first, some alternatives are nominated; then, a single alternative is selected from the set of nominated alternatives.

2.1 The nomination process

We first describe the nomination process of the two-stage voting procedure. Each voter $i \in N$ expresses an *opinion* about X. Voter i's opinion represents his or her view on whether each alternative should be eligible as a candidate for collective choice, and is represented by a $1 \times n$ row vector J_i consisting of 1 or -1. Let J_{ik} denote the kth component of J_i. The interpretation of $J_{ik} = 1$ (respectively, $J_{ik} = -1$) is that voter i has a positive (respectively, negative)

[7] To prove the Erdamar et al. (2017) impossibility result, a sufficient condition for preference domains is that they be circular domains, as proposed by Sato (2010).

view about alternative x_k. Let $\mathscr{J}_i$ be the set of all opinions for voter i. An *opinion profile* J is an $n \times m$ matrix consisting of n row vectors $J_1, \ldots, J_n$. Let $\mathscr{J}$ be the set of all opinion profiles.

In the nomination process, the opinions of voters are aggregated to determine which alternatives are nominated. The process is described as a *nomination rule*, $f : \mathscr{J} \to \mathscr{X}$, which assigns a non-empty subset of X to each opinion profile. For all $J \in \mathscr{J}$, let $f(J)$ denote the set of *nominated alternatives*. That is, $x_k \in f(J)$ if and only if x_k is a nominated alternative under opinion profile J.

2.2 The voting process

We next describe the voting process of the two-stage voting procedure. Each voter $i \in N$ expresses a weak preference over X—that is, a reflexive, complete, and transitive binary relation on X. Let R_i be a weak preference for voter i. Strict preference P_i is defined in the usual way. Let $\mathscr{R}$ be the set of all weak preferences. Given a non-empty subset A of X and a weak preference $R_i \in \mathscr{R}$, $B(A, R_i) = \{x \in A | xR_iy \text{ for all } y \in A\}$ is the set of the best alternatives in A, based on R_i.

We require a consistency condition between each voter's opinion and preference. That is, the set of a voter's preferences, called a *preference space*, is restricted in such a way that his or her preference is consistent with his or her opinion expressed in the nomination process of the two-stage voting procedure. More precisely, it is assumed that every voter always prefers positive to negative alternatives, based on his or her opinion.

Given an opinion profile $J \in \mathscr{J}$, let $\mathcal{D}_i^J \subseteq \mathscr{R}$ be voter i's preference space under opinion profile J. Formally, for all $i \in N$, voter i's preference space $\mathcal{D}_i^J$ is defined as follows. Given an opinion profile $J \in \mathscr{J}$, we define a partition $X_+^{J_i}$ and $X_-^{J_i}$ of X, such that for all $x_k \in X$, $J_{ik} = 1$ implies that x_k is in $X_+^{J_i}$, and $J_{ik} = -1$ implies that x_k is in $X_-^{J_i}$. That is, $X_+^{J_i}$ is the set of positive alternatives for i, and $X_-^{J_i}$ is the set of negative alternatives for i. Voter i's preference space $\mathcal{D}_i^J$ is defined as follows:

$$\mathcal{D}_i^J = \{R_i \in \mathscr{R} | \text{for all } x \in X_+^{J_i} \text{ and all } y \in X_-^{J_i}, xP_iy\}.$$

Given a non-empty subset A of X and an opinion profile $J \in \mathscr{J}$, let $T_i^J(A) = \{x \in A | \exists R_i \in \mathcal{D}_i^J, \text{ such that } x \in B(A, R_i)\}$. That is, $T_i^J(A)$ is the set of all alternatives x in A, such that x is one of the best elements, based on some voter i's preference $R_i \in \mathcal{D}_i^J$. We call $T_i^J(A)$ voter i's *top set of alternatives* in A under J.

For all $J \in \mathscr{J}$ and all $i \in N$, if the two components $X_+^{J_i}$ and $X_-^{J_i}$ of a partition of X are non-empty, then $\mathcal{D}_i^J$ is said to be *dichotomous*. A voter $i \in N$ is a *dichotomist* if $\mathcal{D}_i^J$ is dichotomous. A voter $i \in N$ is an *extreme dichotomist* if he or she is a dichotomist and his or her top set of alternatives in X under J is a singleton; that is, $\#T_j^J(X) = 1$.

A *preference domain* is a Cartesian product of voters' preference spaces $\mathcal{D}^J = \prod_{i \in N} \mathcal{D}_i^J$. Let a *preference profile* $\mathbf{R} = (R_1, \ldots, R_n)$ be n-tuple voters' preferences. Given a nomination rule f and an opinion profile $J \in \mathscr{J}$, we describe the voting process of the two-stage voting procedure as a *voting rule with* $f(J)$, $V_f^J : \mathcal{D}^J \to f(J)$, or simply V_f^J. A voting rule with $f(J)$ is a mapping that assigns an alternative in $f(J)$ to each preference profile $\mathbf{R} \in \mathcal{D}^J$. Let $V_f^J(\mathbf{R}, f(J))$ denote the voting outcome, that is, the alternative chosen from $f(J)$ under $\mathbf{R} \in \mathcal{D}^J$.

For all $J \in \mathscr{J}$, let $\hat{f}(J) = \{x_k \in f(J) | \exists \mathbf{R} \in \mathcal{D}^J, \text{such that } x_k = V_f^J(\mathbf{R}, f(J)\}$. That is, $\hat{f}(J)$ is a set of alternatives x_k in $f(J)$, such that there exists a preference profile $\mathbf{R} \in \mathcal{D}^J$, according to which x_k is the voting outcome chosen from $f(J)$. Clearly, if a voting rule with $f(J)$ is onto, then we have $f(J) = \hat{f}(J)$.

Finally, both the nomination and the voting process of the two-stage voting procedure are defined as a *voting procedure with* f—that is, a family of voting rules with $f(J)$, $\left\{V_f^J : \mathcal{D}^J \to f(J)\right\}_{J \in \mathscr{J}}$, or simply $\{V_f^J\}$.

3 Related literature

3.1 The preference-approval model

Brams and Sanver (2006, 2009) propose the "preference-approval" model, where voters simultaneously express their preferences and evaluations over alternatives. The voters' evaluations divide the set of alternatives into two proper subsets, namely, a set of acceptable alternatives and a set of unacceptable ones. If acceptable alternatives are interpreted as positive and unacceptable alternatives as negative, then the voters' evaluations are technically equivalent to the voters' opinions in the proposed model.

As we assume in the model, each voter's preference is consistent with his or her evaluation in the preference-approval model. For example, Erdamar et al. (2017) assume that if one alternative is at least as good as another in terms of a voter's preference, and the latter alternative is acceptable, then the former alternative is also acceptable. This consistency condition is compatible with that of the proposed model.

Erdamar et al. (2017) consider a consistent preference-approval domain after the preference domain has been restricted exogenously. That is, each voter expresses a consistent pair of a preference and an evaluation under a restricted preference domain. As a result, no voters have evaluations that are inconsistent with their admissible preferences. In contrast, a voter's preference space in the proposed model is restricted endogenously after he or she expresses an opinion in the nomination process. That is, each voter first expresses an opinion, and then expresses a preference that is consistent with that opinion. As a result, no voters have preferences that are inconsistent with their previously stated opinions.

The preference-approval model can be viewed as a one-stage voting procedure because voters simultaneously express their preferences and evaluations, and every alternative is always feasible. Therefore, the preference-approval model does not deal with agenda formation via the nomination process. Agenda formation weakens the axioms in the proposed model. For example, consider an axiom that requires that the voting outcome be a Pareto efficient alternative. Because every alternative is always feasible in the preference-approval model, one-stage voting procedures must satisfy the axiom for all alternatives. However, in the proposed model, only nominated alternatives are feasible in the voting process. Thus, two-stage voting procedures need only satisfy the axiom for the nominated alternatives. This implies that the axioms in the proposed model are weaker than those of the preference-approval model.

3.2 Two-stage collective choice

The two-stage voting procedure was originally proposed by Iwata (2016, 2018) to resolve Arrow's (1963) impossibility theorem. Here, we focus on the strategic aspects of such procedures. As such, we employ a reduced form of the two-stage collective choice of Iwata (2016, 2018) in order to compare our results with those of Erdamar et al. (2017) for the preference-approval model.

The two-stage collective choice of Iwata (2016, 2018) is an extension of the two-stage voting procedures presented here, for the following reasons:

1. Some voters do not always express their opinions;
2. Opinions are trichotomous, rather than dichotomous;
3. No alternative may be nominated in the nomination process;
4. Multiple alternatives may be chosen in the voting process.

In the two-stage collective choice, individuals are nominators (who express their opinions), voters (who express their preferences), or both. Although we assume that every individual is a nominating voter in the model, Iwata (2016, 2018) admits the existence of a voter who is not a nominator, and a nominator who is not a voter.

In addition to positive and negative opinions, Iwata (2016, 2018) allow nominators to express a neutral opinion about an alternative. Therefore, if a voter in his model expresses both opinions and preferences, then his or her preference space is restricted such that he or she prefers positive to neutral alternatives, and neutral to negative alternatives in terms of his or her opinion.

Furthermore, Iwata (2016, 2018) allow the set of nominated alternatives to be empty. In this case, the choice set in the voting process also becomes empty. In contrast, in the proposed model, we assume that at least one alternative is nominated in the nomination process, implying that the set of nominated alternatives is always non-empty. Thus, in our setup, an alternative is always chosen from the set of nominated alternatives.

In the proposed model, the voting process consists of choice functions; in Iwata (2016, 2018), it consists of choice correspondences. Therefore, although the voting outcome must be a single alternative in the proposed model, it is not always a singleton in that of Iwata (2016, 2018).

4 Axioms

4.1 Standard axioms

In this subsection, we introduce the standard axioms in the Arrovian social choice literature. Note that we impose the axioms on the voting procedures with f. Therefore, we define a voting rule with $f(J)$ as satisfying an axiom if the voting procedure with f satisfies the axiom under the given opinion profile $J \in \mathscr{J}$. As we mentioned in Section 3, given an opinion profile $J \in \mathscr{J}$, some standard axioms are imposed for all nominated alternatives, not for all potentially feasible alternatives. This is a crucial difference between the proposed model and the preference-approval model, where the axioms must be imposed for all potentially feasible alternatives.

For all $J \in \mathscr{J}$ and all $\mathbf{R} \in \mathcal{D}^J$, an alternative $x \in f(J)$ is *efficient* if there exists no $y \in f(J)$, such that yR_ix for all $i \in N$, and yP_jx for some $j \in N$. Efficiency requires that an efficient alternative must always be chosen from the set of nominated alternatives.

- **Efficiency:** For all $J \in \mathscr{J}$ and all $\mathbf{R} \in \mathcal{D}^J$, $V_f^J(\mathbf{R}, f(J))$ is an efficient alternative.

Given an opinion profile $J \in \mathscr{J}$, a voter $i \in N$ is *decisive on* $\hat{f}(J)$ if, for all $x, y \in \hat{f}(J)$ and all $\mathbf{R} \in \mathcal{D}^J$, xP_iy implies $y \neq V_f^J(\mathbf{R}, f(J))$. Given an opinion profile $J \in \mathscr{J}$, a voter $i \in N$ is a *dictator on* $f(J)$ if he or she is decisive on $f(J)$. If a voting rule with $f(J)$ is onto and voter i is decisive on $\hat{f}(J)$, then he or she is a dictator on $f(J)$. However, if the voting rule with $f(J)$ is not onto, then voter i is not always a dictator on $f(J)$, even if he or she is decisive on $\hat{f}(J)$.

A voting rule with $f(J)$ is *dictatorial* if a dictator exists on $f(J)$. A voting procedure with f is dictatorial if a dictator exists on $f(J)$ for all $J \in \mathscr{J}$. Non-dictatorship requires that a voting procedure with f not be dictatorial.

- **Non-dictatorship:** No dictator exists on $f(J)$, for all $J \in \mathscr{J}$.

Before we define the next axiom, we introduce some notation and definitions. A permutation of any set is a one-to-one function of that set onto itself. Let π be a permutation of N onto itself. A nomination rule is *anonymous* if, for all $J^1, J^2 \in \mathscr{J}$, the existence of a permutation π with $J^2 = \pi(J^1)$ implies $f(J^1) = f(J^2)$. *Anonymity* requires that all voters be treated symmetrically or equally in both the nomination and the voting process of the voting procedures with f.

- **Anonymity:** For all $J^1, J^2 \in \mathscr{J}$ and all $\mathbf{R}^1, \mathbf{R}^2 \in \mathcal{D}^J$, if there exists a permutation π with $J^2 = \pi(J^1)$ and $\mathbf{R}^2 = \pi(\mathbf{R}^1)$, then $V_f^{J^1}(\mathbf{R}^1, f(J^1)) = V_f^{J^2}(\mathbf{R}^2, f(J^2))$, where f is anonymous.

Thus, by definition, anonymity imposes a restriction not only on the class of voting rules with $f(J)$, but also on the class of nominating rules f. Note that we define a voting rule with $f(J)$ as satisfying anonymity when, for all $\mathbf{R}^1, \mathbf{R}^2 \in \mathcal{D}^J$, if there exists a permutation π with $\mathbf{R}^2 = \pi(\mathbf{R}^1)$, then $V_f^J(\mathbf{R}^1, f(J)) = V_f^J(\mathbf{R}^2, f(J))$.

4.2 Non-manipulability axioms

In this subsection, we introduce the non-manipulability axioms for the voting procedures with f. We first apply the notion of strategy-proofness to the model. Suppose that a voter attempts to change a voting outcome by misrepresenting his or her opinion, preference, or both. If the voting outcome becomes more favorable in terms of his or her preference, then a voting procedure with f is manipulable. Strategy-proofness in the proposed model requires that a voting procedure with f not be manipulable in this sense.

Formally, a voting procedure with f is *manipulable* if there exist $J \in \mathscr{J}$, $\mathbf{R} \in \mathcal{D}^J$, $i \in N$, $J_i' \in \mathscr{J}_i$, and $R_i' \in \mathcal{D}_i^{(J_i', J_{-i})}$, such that

$$V_f^{(J_i', J_{-i})}((R_i', \mathbf{R}_{-i}), f(J_i', J_{-i})) P_i V_f^J(\mathbf{R}, f(J)).$$

- **Strategy-proofness:** A voting procedure with f is not manipulable.

Next, we weaken strategy-proofness in the proposed model. Suppose again that a voter attempts to change a voting outcome by misrepresenting his or her opinion, preference, or both. Note that a voting outcome does not always change from a negative to a positive alternative in terms of the voter's opinion, even when it becomes more favorable in terms of his or her preference. Therefore, if the voting outcome changes from a negative to a positive alternative in terms of his or her opinion, then the gain from such a manipulation is sufficiently large. As a result, the voter has a strong incentive to perform the strategic manipulation. *Opinion-based strategy-proofness* requires that a voting procedure with f not be manipulable only in this strong sense.

Formally, a voting procedure with f is *opinion-based manipulable* if there exist $J \in \mathscr{J}$, $\mathbf{R} \in \mathcal{D}^J$, $i \in N$, $J_i' \in \mathscr{J}_i$, and $R_i' \in \mathcal{D}_i^{(J_i', J_{-i})}$, such that

$$V_f^J(\mathbf{R}, f(J)) \in X_-^{J_i} \text{ and } V_f^{(J_i', J_{-i})}((R_i', \mathbf{R}_{-i}), f(J_i', J_{-i})) \in X_+^{J_i}.$$

- **Opinion-based strategy-proofness:** A voting procedure with f is not opinion-based manipulable.

Note that the opinions of voters in the model are technically equivalent to the evaluations of voters in the preference-approval model. The definitions by Erdamar et al. (2017) of strategy-proofness and its weakening in the preference-approval model are similar to ours, where Erdamar et al. (2017) refer to strategy-proofness and evaluationwise strategy-proofness, respectively.

Finally, we provide a non-manipulability notion for strategic nomination. Suppose that a voter attempts to change a voting outcome by manipulating the set of nominated alternatives under fixed voters' preferences. Given any other voters' opinions, he or she might add new elements to the set of nominated alternatives or remove candidates from the set by changing his or her opinions. *Stability* requires that the voting outcome remains the same if it is still nominated even after his or her strategic nomination.

- **Stability:** For all $J \in \mathscr{J}$, $i \in N$, $J'_i \in \mathscr{J}_i$, and $\mathbf{R} \in \mathcal{D}^J \cap \mathcal{D}^{(J'_i, J_{-i})}$, if $V_f^J(\mathbf{R}, f(J)) \in f(J'_i, J_{-i})$, then $V_f^J(\mathbf{R}, f(J)) = V_f^{(J'_i, J_{-i})}(\mathbf{R}, f(J'_i, J_{-i}))$.

The voting outcome clearly does not remain the same if it is not nominated when a voter changes his or her opinions. Formally, for all $J \in \mathscr{J}$, $i \in N$, $J'_i \in \mathscr{J}_i$, and $\mathbf{R} \in \mathcal{D}^J \cap \mathcal{D}^{(J'_i, J_{-i})}$, if we have $V_f^J(\mathbf{R}, f(J)) \notin f(J'_i, J_{-i})$, then $V_f^J(\mathbf{R}, f(J)) \neq V_f^{(J'_i, J_{-i})}(\mathbf{R}, f(J'_i, J_{-i}))$, by definition of the voting procedures with f.

We now provide preliminary results for strategy-proof voting procedures with f in the model. We focus on a specific opinion profile, such that every voter has a positive opinion about each alternative. Let $J^* \in \mathscr{J}$ be the opinion profile such that $J_{ik} = 1$, for all $i \in N$ and $x_k \in X$.

Lemma 1 shows that if a voting procedure with f is strategy-proof, then every voting outcome must always be chosen from the set of alternatives that could be chosen from $f(J^*)$.

Lemma 1. *If a voting procedure with f is strategy-proof, then for all $J \in \mathscr{J}$ and all $\boldsymbol{R} \in \mathcal{D}^J$, $V_f^J(\boldsymbol{R}, f(J)) \in \hat{f}(J^*)$.*

Proof. Let $J \in \mathscr{J}$ be any opinion profile. Let $J^1, J^2, \ldots, J^n$ be a sequence of opinion profiles, such that $J^1 = (J_1, J^*_{-1})$, $J^2 = (J_2, J^1_{-2})$, …, $J^n = (J_n, J^{n-1}_{-n})$. Note that $J^n = J$. Consider J^* and J^1. Suppose there exists $\mathbf{R} \in \mathcal{D}^{J^1}$, such that $V_f^{J^1}(\mathbf{R}, f(J^1)) \notin \hat{f}(J^*)$. Because $\mathcal{D}_1^{J^*}$ is unrestricted, there exists $R_1 \in \mathcal{D}_1^{J^*}$, such that $V_f^{J^1}(\mathbf{R}, f(J^1)) P_1 V_f^{J^*}(R_1, \mathbf{R}_{-1}, f(J^*))$, which is a contradiction to strategy-proofness. Therefore, we have $V_f^{J^1}(\mathbf{R}, f(J^1)) \in \hat{f}(J^*)$, for all $\mathbf{R} \in \mathcal{D}^{J^1}$.

Using a similar argument, we can show that for all $t \in \{2, \ldots, n\}$, $V_f^{J^t}(\mathbf{R}, f(J^t)) \in \hat{f}(J^*)$, for all $\mathbf{R} \in \mathcal{D}^{J^t}$, which implies that $V_f^J(\mathbf{R}, f(J)) \in \hat{f}(J^*)$, for all $J \in \mathscr{J}$ and all $\mathbf{R} \in \mathcal{D}^J$. $\square$

Next, Lemma 2 considers the following situation. Suppose $f(J^*)$ contains only two alternatives and that a voting procedure with f is strategy-proof. Then, if no voter is decisive on $\hat{f}(J^*)$, the alternatives in $\hat{f}(J^*)$ must always be nominated.

Lemma 2. *Suppose that a voting procedure with f is strategy-proof and $\#f(J^*) = 2$. If no voter is decisive on $\hat{f}(J^*)$, then for all $J \in \mathscr{J}$, $\hat{f}(J^*) \subseteq f(J)$.*

Proof. If $\#\hat{f}(J^*) = 1$, then every voter is decisive on $\hat{f}(J^*)$, which is a contradiction. Therefore, we must have $\hat{f}(J^*) = f(J^*)$, because of $\#f(J^*) = 2$.

Let $\hat{f}(J^*) = \{x_k, x_l\}$. Given any opinion profile $J \in \mathscr{J}$, let $J^1, J^2, \ldots, J^n$ be a sequence of opinion profiles, such that $J^1 = (J_1, J^*_{-1})$, $J^2 = (J_2, J^1_{-2}), \ldots$, $J^n = (J_n, J^{n-1}_{-n})$. Note that we have $J^n = J$.

Consider J^* and J^1. Without loss of generality, let $x_k \in \hat{f}(J^*) \backslash f(J^1)$. Because voter 1 is not decisive on $\hat{f}(J^*)$, there exists $\mathbf{R} \in \mathcal{D}^{J^*}$, such that $x_l P_1 x_k$ and $V_f^{J^*}(\mathbf{R}, f(J^*)) = x_k$. By Lemma 1, we have $V_f^{J^1}(\mathbf{R}, f(J^1)) \in \hat{f}(J^*)$, for all $\mathbf{R} \in \mathcal{D}^{J^1}$. Because we have $x_k \in \hat{f}(J^*) \backslash f(J^1)$, we must have $x_l = V_f^{J^1}(R'_1, \mathbf{R}_{-1}, f(J^1))$, for all $R'_1 \in \mathcal{D}_1^{J^1}$. This implies that $V_f^{J^1}(R'_1, \mathbf{R}_{-1}, f(J^1)) P_1 V_f^{J^*}(\mathbf{R}, f(J^*))$, which is a contradiction to strategy-proofness. Therefore, we have $\hat{f}(J^*) \subseteq f(J^1)$. Using a similar argument, we have $\hat{f}(J^*) \subseteq f(J^t)$, for all $t \in \{2, \ldots, n\}$, which completes the proof. $\square$

Lemma 3 shows that if a voting procedure with f is strategy-proof and a voter exists who is decisive on $\hat{f}(J^*)$, then he or she is a dictator on $f(J)$, for all $J \in \mathscr{J} \backslash \{J^*\}$, and his or her top set of alternatives in $f(J)$ must always be the same as that of $\hat{f}(J^*)$.

Lemma 3. *Suppose that a voting procedure with f is strategy-proof. If there exists a voter i who is decisive on $\hat{f}(J^*)$, then for all $J \in \mathscr{J} \backslash \{J^*\}$, voter i is a dictator on $f(J)$ and $T_i^J(\hat{f}(J^*)) = T_i^J(f(J))$.*

Proof. Let a voter i be decisive on $\hat{f}(J^*)$. Given any opinion profile $J \in \mathscr{J} \backslash \{J^*\}$, let $J^1, J^2, \ldots, J^n$ be a sequence of opinion profiles, such that $J^1 = (J_1, J^*_{-1}), \ldots, J^{i-1} = (J_{i-1}, J^{i-1}_{-(i-1)})$, $J^i = (J_{i+1}, J^{i-1}_{-(i+1)}), \ldots, J^{n-1} = (J_n, J^{n-2}_{-n})$, and $J^n = (J_i, J^{n-1}_{-i})$. Note that we have $J^n = J$.

Consider J^* and J^1. We first show the following result.

Result 1. *$\hat{f}(J^*) = f(J^1)$ and voter i is a dictator on $f(J^1)$.*

Proof. First, we show $\hat{f}(J^*) \subseteq f(J^1)$. Suppose there exists $x_k \in \hat{f}(J^*) \backslash f(J^1)$. Because $\mathcal{D}_1^{J^*}$ and $\mathcal{D}_i^{J^*}$ are unrestricted, there exists $\mathbf{R} \in \mathcal{D}^{J^*}$, such that $\{x_k\} = B(\hat{f}(J^*), R_i)$ and $x_l P_1 x_k$, for all $x_l \in \hat{f}(J^*) \backslash \{x_k\}$. Because voter i is decisive on $\hat{f}(J^*)$, we have $x_k = V_f^{J^*}(\mathbf{R}, f(J^*))$. From Lemma 1, we have $V_f^{J^1}(R'_1, \mathbf{R}_{-1}, f(J^1)) \in \hat{f}(J^*)$, for all $R'_1 \in \mathcal{D}_1^{J^1}$. This implies that $V_f^{J^1}(R'_1, \mathbf{R}_{-1}, f(J^1)) P_1 V_f^{J^*}(\mathbf{R}, f(J^*))$, which is a contradiction to strategy-proofness. Thus, we must have $\hat{f}(J^*) \subseteq f(J^1)$.

Second, we show that voter i is a dictator on $f(J^1)$. Suppose that voter i is not a dictator on $f(J^1)$. Then, there exist $x_k, x_l \in f(J^1)$ and $\mathbf{R} \in \mathcal{D}^{J^1}$, such that $x_k P_i x_l$ and $x_l = V_f^{J^1}(\mathbf{R}, f(J^1))$. Because $\mathcal{D}^{J^*}$ is unrestricted, there exists $R'_1 \in \mathcal{D}_1^{J^*}$, such that $x_l P'_1 x^*$, where $x^* \in B(\hat{f}(J^*), R_i)$. Because i is decisive on $\hat{f}(J^*)$, there exists an alternative x^*, such that $x^* = V_f^{J^*}(R'_1, \mathbf{R}_{-1}, f(J^*))$. This implies that $V_f^{J^1}(\mathbf{R}, f(J^1)) P'_1 V_f^{J^*}(R'_1, \mathbf{R}_{-1}, f(J^*))$, which is a contradiction to strategy-proofness. Thus, voter i must be a dictator on $f(J^1)$.

Third, we show $f(J^1) \subseteq \hat{f}(J^*)$. Suppose there exists $x_k \in f(J^1)\backslash\hat{f}(J^*)$. Because $\mathcal{D}_i^{J^1}$ is unrestricted, there exists $\mathbf{R} \in \mathcal{D}^{J^1}$, such that $\{x_k\} = B(f(J^1), R_i)$. From the dictatorship of i on $f(J^1)$, we have $x_k = V_f^{J^1}(\mathbf{R}, f(J^1))$, which is a contradiction to Lemma 1. Thus, we must have $f(J^1) \subseteq \hat{f}(J^*)$, and we have the required result. $\square$

By repeatedly using a similar argument to that for Result 1, we have $\hat{f}(J^*) = f(J^t)$ and voter i is a dictator on $f(J^t)$, for all $t \in \{2, \ldots, n-1\}$.

Now, we show that voter i is a dictator on $f(J^n)$ and $T_i^{J^n}(\hat{f}(J^*)) = T_i^{J^n}(f(J^n))$. First, we show $T_i^{J^n}(\hat{f}(J^*)) \subseteq f(J^n)$. Suppose there exists $x_k \in T_i^{J^n}(\hat{f}(J^*))\backslash f(J^n)$. Because we have $x_k \in T_i^{J^n}(\hat{f}(J^*))$, there exists $\mathbf{R} \in \mathcal{D}^{J^n}$, such that $\{x_k\} = B(\hat{f}(J^*), R_i)$ and $x_k \neq V_f^{J^n}(\mathbf{R}, f(J^n))$. From Lemma 1, we have $V_f^{J^n}(\mathbf{R}, f(J^n)) \in \hat{f}(J^*)$. Note that voter i is a dictator on $f(J^{n-1})$ and $\hat{f}(J^*) = f(J^{n-1})$. Because $\mathcal{D}_i^{J^{n-1}}$ is unrestricted, there exists $R_i' \in \mathcal{D}_i^{J^{n-1}}$, such that $\{x_k\} = B(f(J^{n-1}), R_i')$. From the dictatorship of i on $f(J^{n-1})$, we have $x_k = V_f^{J^{n-1}}(R_i', \mathbf{R}_{-i}, f(J^{n-1}))$. This implies that $V_f^{J^{n-1}}(R_i', \mathbf{R}_{-i}, f(J^{n-1})) P_i V_f^{J^n}(\mathbf{R}, f(J^n))$, which contradicts strategy-proofness. Thus, we have $T_i^{J^n}(\hat{f}(J^*)) \subseteq f(J^n)$.

Next, we show that voter i is a dictator on $f(J^n)$. Suppose that i is not a dictator on $f(J^n)$. Then, there exist $x_k, x_l \in f(J^n)$ and $\mathbf{R} \in \mathcal{D}^{J^n}$, such that $x_k P_i x_l$ and $x_l = V_f^{J^n}(\mathbf{R}, f(J^n))$. From Lemma 1, we have $V_f^{J^n}(\mathbf{R}, f(J^n)) \in \hat{f}(J^*)$. Because $\mathcal{D}_i^{J^{n-1}}$ is unrestricted, there exists $R_i' \in \mathcal{D}_i^{J^{n-1}}$, such that $\{x_k\} = B(f(J^{n-1}), R_i')$. From the dictatorship of i on $f(J^{n-1})$, we have $x_k = V_f^{J^{n-1}}(R_i', \mathbf{R}_{-i}, f(J^{n-1}))$. This implies that $V_f^{J^{n-1}}(R_i', \mathbf{R}_{-i}, f(J^{n-1})) P_i V_f^{J^n}(\mathbf{R}, f(J^n))$, which is a contradiction to strategy-proofness. Thus, voter i must be a dictator on $f(J^n)$.

Finally, we show $T_i^{J^n}(\hat{f}(J^*)) = T_i^{J^n}(f(J^n))$. Suppose there exists $x_k \in T_i^{J^n}(f(J^n))\backslash T_i^{J^n}(\hat{f}(J^*))$. Because we have $T_i^{J^n}(\hat{f}(J^*)) \subseteq f(J^n)$, $x_k \notin \hat{f}(J^*)$ must hold. Consider $\mathbf{R} \in \mathcal{D}^{J^n}$, with $\{x_k\} = B(f(J^n), R_i)$. From the dictatorship of i on $f(J^n)$, we have $x_k = V_f^{J^n}(\mathbf{R}, f(J^n))$, which is a contradiction to Lemma 1. Therefore, we have $T_i^{J^n}(f(J^n)) \subseteq T_i^{J^n}(\hat{f}(J^*))$.

Furthermore, suppose there exists $x_k \in T_i^{J^n}(\hat{f}(J^*))\backslash T_i^{J^n}(f(J^n))$. Because we have $T_i^{J^n}(\hat{f}(J^*)) \subseteq f(J^n)$, we must have $\hat{f}(J^*) \cap T_i^{J^n}(f(J^n)) = \emptyset$. For some $x_l \in T_i^{J^n}(f(J^n))$, consider $\mathbf{R} \in \mathcal{D}^{J^n}$, with $\{x_l\} = B(f(J^n), R_i)$. From the dictatorship of i on $f(J^n)$, we have $x_l = V_f^{J^n}(\mathbf{R}, f(J^n))$. However, we have $x_l \notin \hat{f}(J^*)$, which is a contradiction to Lemma 1. Therefore, we have $T_i^{J^n}(\hat{f}(J^*)) \subseteq T_i^{J^n}(f(J^n))$. $\square$

As a corollary to Lemma 3, a voting procedure with f may not always be strategy-proof, even if it is dictatorial. This is because strategy-proofness in the model restricts both the class of voting rules with $f(J)$ and the class of nomination rules f. If a dictator exists on $f(J)$, for all $J \in \mathscr{J}$, then no voter has an incentive to manipulate the voting outcome by misrepresenting only his or her preference. However, he or she may be able to manipulate the outcome by misrepresenting his or her opinion.

For example, suppose that voter i is a dictator on $f(J)$, for all $J \in \mathscr{J}$. Consider a nomination rule under which there exists a voter j, such that if he or she has a positive opinion about any alternative x_k, then x_k is nominated; that is, for all $J \in \mathscr{J}$ and $x_k \in X$, if $J_{jk} = 1$, then $x_k \in f(J)$.[8]

Let $X = \{x_1, x_2, x_3\}$. Consider an opinion profile $J \in \mathscr{J}$, such that $J_{j1} = J_{j2} = J_{j3} = 1$. By definition, we have $f(J) = X$. Consider a preference profile $\mathbf{R} \in \mathcal{D}^J$, such that $\{x_1\} = B(f(J), R_i)$ and $x_3 P_j x_1$. From the dictatorship of i on $f(J)$, we have $x_1 = V_f^J(\mathbf{R}, f(J))$. Consider voter j's opinion $J'_j \in \mathscr{J}_j$, such that $J_{j1} = J_{j2} = -1$ and $J_{j3} = 1$. By construction of f, we have $f(J'_j, J_{-j}) = \{x_3\}$. Therefore, we have $V_f^{(J'_j, J_{-j})}(R'_j, \mathbf{R}_{-j}, f(J'_j, J_{-j})) P_j V_f^J(\mathbf{R}, f(J))$, for any $R'_j \in \mathcal{D}_j^{(J'_j, J_{-j})}$, which implies that the voting procedure with f is manipulable.

In the next corollary, we summarize the above observations.

Corollary 1. *There exists a dictatorial and manipulable voting procedure with f.*

5 Main theorems

In this section, we provide our main theorems. We first characterize the properties of strategy-proof voting procedures with f. Our results show that strategy-proofness is a demanding axiom in the proposed model that drastically restricts the class of voting procedures with f. Therefore, we weaken strategy-proofness to opinion-based strategy-proofness, and attempt to find a voting procedure with f that is stable and opinion-based strategy-proof.

In the preference-approval model, Erdamar et al. (2017) propose an invariance axiom related to the approvals of voters. This axiom requires that voting outcomes be independent of the voters' evaluations, and plays an essential role in reducing the preference-approval model to the standard Arrovian social choice model. The next property, *opinion invariance*, is a counterpart to the invariance axiom of Erdamar et al. (2017) for the preference-approval model.

- **Opinion invariance:** For all $J, J' \in \mathscr{J}$ and $\mathbf{R} \in \mathcal{D}^J \cap \mathcal{D}^{J'}$, $V_f^J(\mathbf{R}, f(J)) = V_f^{J'}(\mathbf{R}, f(J'))$.

Note that opinion invariance is an unreasonable property in the proposed model, because it implies that the voting outcome is always chosen from the intersection of any two sets of nominated alternatives; that is, for all $J, J' \in \mathscr{J}$ and $\mathbf{R} \in \mathcal{D}^J \cap \mathcal{D}^{J'}$, $V_f^J(\mathbf{R}, f(J)) = V_f^{J'}(\mathbf{R}, f(J')) = x_k$ implies $x_k \in f(J) \cap f(J')$. If the intersection of two sets of nominated alternatives is empty, then every voting procedure with f violates opinion invariance. However, because the set

[8] If voter j does not have a positive opinion about any alternatives, then we suppose every alternative is nominated. However, this assumption is not crucial in the example.

of nominated alternatives varies with the voters' opinions, opinion invariance restricts the class of nomination rules in an undesirable way.

Because every alternative is always feasible in the preference-approval model, the evaluation invariance axiom of Erdamar et al. (2017) can be viewed as reducing the preference-approval model to the standard Arrovian social choice model with a fixed agenda. In contrast, the following property reduces the model to an Arrovian social choice model with multiple agendas. The voting outcome does not change if the set of nominated alternatives remains the same after the voters' opinions change.[9] Formally, for all $J, J' \in \mathscr{J}$ and $\mathbf{R} \in \mathcal{D}^J \cap \mathcal{D}^{J'}$, if $f(J) = f(J')$, then $V_f^J(\mathbf{R}, f(J)) = V_f^{J'}(\mathbf{R}, f(J'))$. This property is clearly implied by opinion invariance. Thus, opinion invariance is too demanding to provide a meaningful interpretation in the proposed model.

However, as shown in Theorem 1, if a voting procedure with f is strategy-proof and stable, then it must be opinion invariant.[10] This implies that we have to exclude either strategy-proofness or stability from the list of axioms in the proposed model. In addition, Theorem 1 shows that if a voting procedure with f is opinion-based strategy-proof and opinion invariant, then it is strategy-proof and stable.

Theorem 1. *A voting procedure with f is strategy-proof and stable if and only if it is opinion-based strategy-proof and opinion invariant.*

Proof. Opinion invariance clearly implies stability.

We first show that if a voting procedure with f is strategy-proof and stable, then it is opinion invariant. We identify three possible cases: (i) $\#\hat{f}(J^*) = 1$; (ii) $\#\hat{f}(J^*) = 2$, and no voter i is decisive on $\hat{f}(J^*)$; and (iii) all other cases.

Case (i). Let $\{x_k\} = \hat{f}(J^*)$. From Lemma 1, we have $V_f^J(\mathbf{R}, f(J)) \in \hat{f}(J^*)$, for all $J \in \mathscr{J}$ and $\mathbf{R} \in \mathcal{D}^J$, which implies that $x_k = V_f^J(\mathbf{R}, f(J))$. Thus, the voting procedure with f is opinion invariant in Case (i).

Case (ii). Given any opinion profile $J \in \mathscr{J}$, let $J^1, J^2, \ldots, J^n$ be a sequence of opinion profiles, such that $J^1 = (J_1, J^*_{-1})$, $J^2 = (J_2, J^1_{-2})$, $\ldots$, $J^n = (J_n, J^{n-1}_{-n})$. Note that we have $J^n = J$.

Consider J^* and J^1. From Lemma 2, we have $\hat{f}(J^*) \subseteq f(J^1)$. From Lemma 1, we have $V_f^{J^1}(\mathbf{R}, f(J^1)) \in \hat{f}(J^*)$ for all $\mathbf{R} \in \mathcal{D}^{J^1}$. From the stability, we have $V_f^{J^*}(\mathbf{R}, f(J^*)) = V_f^{J^1}(\mathbf{R}, f(J^1))$, for all $\mathbf{R} \in \mathcal{D}^{J^1}$, with $\mathcal{D}^{J^1} \subseteq \mathcal{D}^{J^*}$. Using a similar argument, for all $t \in \{2, \ldots, n\}$, we have $V_f^{J^{t-1}}(\mathbf{R}, f(J^{t-1})) = V_f^{J^t}(\mathbf{R}, f(J^t))$, for all $\mathbf{R} \in \mathcal{D}^{J^t}$, with $\mathcal{D}^{J^t} \subseteq \mathcal{D}^{J^{t-1}}$, which implies that $V_f^J(\mathbf{R}, f(J)) = V_f^{J^*}(\mathbf{R}, f(J^*))$ for all $\mathbf{R} \in \mathcal{D}^J$. Therefore, we have $V_f^J(\mathbf{R}, f(J)) = V_f^{J^*}(\mathbf{R}, f(J^*)) = V_f^{J'}(\mathbf{R}, f(J'))$, for all $J, J' \in \mathscr{J}$ and $\mathbf{R} \in \mathcal{D}^J \cap \mathcal{D}^{J'}$, which implies that the voting procedure with f is opinion invariant in Case (ii).

[9] This property is originally mentioned in Iwata (2016), and is called nominee invariance by Iwata (2018).

[10] In the preference-approval model, Erdamar et al. (2017) show a similar result to Theorem 1. However, instead of stability, they use a restricted evaluation invariance axiom that is both technical and has no natural interpretation in the model.

Case (iii). We have $\#\hat{f}(J^*) \geq 3$ or $\#\hat{f}(J^*) = 2$, and a voter i exists who is decisive on $\hat{f}(J^*)$. Let $\#\hat{f}(J^*) \geq 3$. Consider the voting rule with $f(J^*)$, whose range is restricted from $f(J^*)$ to $\hat{f}(J^*)$, $\hat{f} : \mathcal{D}^{J^*} \to \hat{f}(J^*)$. By definition, it is clear that the restricted voting rule with $\hat{f}(J^*)$ is onto and strategy-proof in the Gibbard–Satterthwaite sense. In addition, the preference domain $\mathcal{D}^{J^*}$ is unrestricted. Therefore, we can apply the Gibbard–Satterthwaite theorem to the restricted voting rule with $\hat{f}(J^*)$. Thus, there exists a voter i who is decisive on $\hat{f}(J^*)$ in the case of $\#\hat{f}(J^*) \geq 3$.

Given any opinion profile $J \in \mathscr{J}$, let $J^1, J^2, \ldots, J^n$ be a sequence of opinion profiles, such that $J^1 = (J_1, J^*_{-1}), \ldots, J^{i-1} = (J_{i-1}, J^{i-1}_{-(i-1)})$, $J^i = (J_{i+1}, J^{i-1}_{-(i+1)})$, $\ldots$, $J^{n-1} = (J_n, J^{n-2}_{-n})$, and $J^n = (J_i, J^{n-1}_{-i})$. Note that we have $J^n = J$.

Consider J^* and J^1. From Result 1, we have $\hat{f}(J^*) = f(J^1)$ and voter i is a dictator on $f(J^1)$. For all $\mathbf{R} \in \mathcal{D}^{J^1} \subseteq \mathcal{D}^{J^*}$, we have $V_f^{J^*}(\mathbf{R}, f(J^*)) \in B(\hat{f}(J^*), R_i)) \subseteq f(J^1)$, from the decisiveness of i on $f(J^*)$. From the stability, we have $V_f^{J^*}(\mathbf{R}, f(J^*)) = V_f^{J^1}(\mathbf{R}, f(J^1))$. Using a similar argument, we have $V_f^{J^{t-1}}(\mathbf{R}, f(J^{t-1})) = V_f^{J^t}(\mathbf{R}, f(J^t))$, for all $t \in \{2, \ldots, n-1\}$ and all $\mathbf{R} \in \mathcal{D}^{J^t} \subseteq \mathcal{D}^{t^{t-1}}$.

From Lemma 3, voter i is a dictator on $f(J^n)$, and we have $T_i^{J^n}(\hat{f}(J^*)) = T_i^{J^n}(f(J^n)) \subseteq f(J^{n-1})$. For all $\mathbf{R} \in \mathcal{D}^{J^n} \subseteq \mathcal{D}^{J^{n-1}}$, we have $V_f^{J^n}(\mathbf{R}, f(J^n)) \in B(f(J^n), R_i)) \subseteq f(J^{n-1})$, from the dictatorship of i on $f(J^n)$. From the stability, we have $V_f^{J^{n-1}}(\mathbf{R}, f(J^{n-1})) = V_f^{J^n}(\mathbf{R}, f(J^n))$.

The above arguments imply that we have $V_f^{J^n}(\mathbf{R}, f(J^n)) = V_f^{J^*}(\mathbf{R}, f(J^*))$, for all $\mathbf{R} \in \mathcal{D}^{J^n} \subseteq \mathcal{D}^{J^*}$. Therefore, we have $V_f^{J}(\mathbf{R}, f(J)) = V_f^{J^*}(\mathbf{R}, f(J^*)) = V_f^{J'}(\mathbf{R}, f(J'))$, for all $J, J' \in \mathscr{J}$ and all $\mathbf{R} \in \mathcal{D}^J \cap \mathcal{D}^{J'}$, which implies that the voting procedure with f is opinion invariant in Case (iii).

Next, we show that if a voting procedure with f is strategy-proof, then it is also opinion-based strategy-proof. Let $J \in \mathscr{J}$, $\mathbf{R} \in \mathcal{D}^J$, $i \in N$, $J'_i \in \mathscr{J}_i$, and $R'_i \in \mathcal{D}_i^{(J'_i, J_{-i})}$. Let $V_f^J(\mathbf{R}, f(J)) \in X_-^{J_i}$. Suppose $V_f^{(J'_i, J_{-i})}(R'_i, \mathbf{R}_{-i}, f(J'_i, J_{-i})) \notin X_-^{J_i}$. We have $V_f^{(J'_i, J_{-i})}(R'_i, \mathbf{R}_{-i}, f(J'_i, J_{-i})) P_i V_f^J(\mathbf{R}, f(J))$, by definition, which is a contradiction to strategy-proofness.

Finally, we show that if a voting procedure with f is opinion-based strategy-proof and opinion invariant, then it is strategy-proof. Let $J \in \mathscr{J}$, $\mathbf{R} \in \mathcal{D}^J$, $i \in N$, $J'_i \in \mathscr{J}_i$, and $R'_i \in \mathcal{D}_i^{(J'_i, J_{-i})}$. Let $V_f^J(\mathbf{R}, f(J)) = x_k$. Let J''_i be such that $J''_{ik} = -1$, and for all $x_l \in X \backslash \{x_k\}$, if $x_l P_i x_k$, then $J''_{il} = 1$, and $J''_{il} = -1$, otherwise. Note that we have $\mathbf{R} \in \mathcal{D}^{(J''_i, J_{-i})}$. From the opinion invariance, we have $x_k = V_f^{(J''_i, J_{-i})}(\mathbf{R}, f(J''_i, J_{-i}))$. It follows from opinion-based strategy-proofness that $V_f^{(J'_i, J_{-i})}(R'_i, \mathbf{R}_{-i}, f(J'_i, J_{-i})) \notin X_+^{J''_i}$ holds. Because $V_f^{(J''_i, J_{-i})}(\mathbf{R}, f(J''_i, J_{-i}))$ is one of the best alternatives in $X_-^{J''_i}$, according to R_i, we have $V_f^{(J''_i, J_{-i})}(\mathbf{R}, f(J''_i, J_{-i})) R_i V_f^{(J'_i, J_{-i})}(R'_i, \mathbf{R}_{-i}, f(J'_i, J_{-i}))$. Because we have $V_f^J(\mathbf{R}, f(J)) = V_f^{(J''_i, J_{-i})}(\mathbf{R}, f(J''_i, J_{-i})) = x_k$, we obtain $V_f^J(\mathbf{R}, f(J)) R_i V_f^{(J'_i, J_{-i})}(R'_i, \mathbf{R}_{-i}, f(J'_i, J_{-i}))$, which implies that the voting procedure with f is strategy-proof. $\square$

Next, Theorem 2 shows that a necessary and sufficient condition for an efficient and non-dictatorial voting procedure with f to be strategy-proof is that there are two elements in $f(J^*)$.

Theorem 2. *There exists a voting procedure with f that is efficient, non-dictatorial, and strategy-proof if and only if $\#f(J^*) = 2$.*

Proof. We first show that if $\#f(J^*) = 2$, then there exists a voting procedure with f that is efficient, non-dictatorial, and strategy-proof. Consider a constant nomination rule. That is, for all $J, J' \in \mathscr{J}$, $f(J) = f(J')$. Let $f(J^*) = \{x_k, x_l\}$. Consider the simple majority rule with the priority to x_k over x_l. That is, for all $J \in \mathscr{J}$ and $\mathbf{R} \in \mathcal{D}^J$, $x_k = V_f^J(\mathbf{R}, f(J))$ if and only if $\#\{i \in N | x_k P_i x_l\} \geq \#\{i \in N | x_l P_i x_k\}$. The voting procedure with f is clearly efficient, non-dictatorial, and strategy-proof.

Next, suppose that a voting procedure with f is efficient, non-dictatorial, and strategy-proof. Because the preference domain $\mathcal{D}^{J^*}$ is unrestricted, we have $f(J^*) = \hat{f}(J^*)$, by efficiency. Because the voting procedure with f is strategy-proof, we have $V_f^J(\mathbf{R}, f(J)) \in f(J^*)$, for all $J \in \mathscr{J}$ and all $\mathbf{R} \in \mathcal{D}^J$, by Lemma 1. If $\#f(J^*) = 1$, then the voting procedure with f is dictatorial, which is a contradiction.

Let $\#f(J^*) \geq 3$. Note that $\mathcal{D}^{J^*}$ is unrestricted and the voting rule with $f(J^*)$ is onto, by efficiency. In addition, the voting rule with $f(J^*)$ is strategy-proof in the Gibbard–Satterthwaite sense. Therefore, the voting rule with $f(J^*)$ is dictatorial, by the Gibbard–Satterthwaite theorem. We now show that the voting procedure with f is dictatorial. Let $i \in N$ be a dictator on $f(J^*)$. Thus, we show that voter i is a dictator on $f(J)$ for all $J \in \mathscr{J}$.

Consider the following sequence of opinion profiles $J^1, \ldots, J^n$: $J^1 = (J_1, J^*_{-1})$, $\ldots$, $J^{i-1} = (J_{i-1}, J^{i-1}_{-(i-1)})$, $J^i = (J_{i+1}, J^{i-1}_{-(i+1)})$, $\ldots$, $J^{n-1} = (J_n, J^{n-2}_{-n})$, and $J^n = (J_i, J^{n-1}_{-i})$. Note that we have $J = J^n$.

Consider J^* and J^1. From Lemma 1, we have $V_f^{J^1}(\mathbf{R}, f(J^1)) \in f(J^*)$. From Result 1, voter i is a dictator on $f(J^1)$ and $f(J^*) = f(J^1)$. Using a similar argument, for all $t \in \{2, \ldots, n-1\}$, we have $V_f^{J^t}(\mathbf{R}, f(J^t)) \in f(J^*)$, and voter i is a dictator on $f(J^t)$ and $f(J^*) = f(J^t)$. From Lemmas 1 and 3, we have $V_f^{J^n}(\mathbf{R}, f(J^n)) \in f(J^*)$, voter i is a dictator on $f(J^n)$, and $T_i^{J^n}(f(J^*)) = T_i^{J^n}(f(J^n))$. However, this implies that the voting procedure with f is dictatorial, which is a contradiction. $\square$

As a corollary to Theorem 2 and Lemma 3, we show that a Gibbard–Satterthwaite-type impossibility is still valid in the proposed model.

Corollary 2. *If $\#f(J^*) \geq 3$, then every efficient and strategy-proof voting procedure with f is dictatorial, and for all $J \in \mathscr{J}$, $T_i^J(f(J^*)) = T_i^J(f(J))$ holds, where voter i is a dictator on $f(J)$.*

Corollary 2 has stronger implications than the Gibbard–Satterthwaite theorem in the standard Arrovian social choice model. In our terminology, the Gibbard–Satterthwaite theorem states that given an opinion profile $J \in \mathscr{J}$, every efficient and strategy-proof voting rule with $f(J)$ is dictatorial if $\#f(J) \geq 3$ and $\mathcal{D}^J$ is unrestricted. Therefore, the theorem is a result about voting rules with $f(J)$. In contrast, Corollary 2 is a result about this and about nomination rules f. Thus, if $\#f(J^*) \geq 3$ and a voting procedure with f is efficient and strategy-proof, then there exists a dictator on $f(J)$ for all $J \in \mathscr{J}$. Furthermore, the nomination rule f is under the dictator's control in that his or her top set of alternatives in $f(J)$ under J must be equivalent to his or her top set of alternatives in $f(J^*)$ under J.

As shown in Theorems 1 and 2, strategy-proofness in the model drastically restricts both the class of nomination rules f and the class of voting rules with $f(J)$. Next, we identify a voting procedure with f that satisfies the standard axioms and stability when strategy-proofness is weakened to opinion-based strategy-proofness. Theorem 3 shows that the answer is positive.

Before we provide the formal statement of Theorem 3, we introduce some additional notation. For all $J \in \mathscr{J}$ and $x_k \in X$, define the number of voters who have positive opinions about x_k as $N_+^J(x_k) = \#\{i \in N | x_k \in X_+^{J_i}\}$. Let $OPl(J) = \{x_k \in X | \forall x_l \in X,\ N_+^J(x_k) \geq N_+^J(x_l)\}$. That is, $OPl(J)$ is the set of alternatives x_k, such that x_k is a positive alternative for the greatest number of voters. We call $OPl(J)$ the *opinion-based plurality set of alternatives under* J.

For all $J \in \mathscr{J}$, $\mathbf{R} \in \mathcal{D}^J$, and $x_k \in X$, define the number of voters such that x_k is one of the best alternatives in $f(J)$ according to R_i as $N^{\mathbf{R}}(x_k, f(J)) = \#\{i \in N | x_k \in B(f(J), R_i)\}\}$. Let $Pl(f(J), \mathbf{R}) = \{x_k \in f(J) | \forall x_l \in f(J), N^{\mathbf{R}}(x_k, f(J)) \geq N^{\mathbf{R}}(x_l, f(J))\}$. That is, $Pl(f(J), \mathbf{R})$ is the set of alternatives x_k in $f(J)$ such that x_k is one of the best alternatives for the greatest number of voters. We call $Pl(f(J), \mathbf{R})$ the *plurality set of alternatives in* $f(J)$ *under* $\mathbf{R}$.

We are now ready to provide Theorem 3.

Theorem 3. *There exists a voting procedure with* f *that is efficient, non-dictatorial, anonymous, opinion-based strategy-proof, and stable.*

Proof. Consider the following nomination rule f: For all $J \in \mathscr{J}$, (a) if every voter $i \in N$ is an extreme dichotomist, $f(J)$ consists of the alternatives x_k such that a voter $i \in N$ exists who has $\{x_k\} = T_i^J(X)$; and (b) if at least one voter i is not an extreme dichotomist, then $f(J)$ consists of only the first alternative $x_k \in OPl(J)$ according to the numerical order.

Construct the following voting procedure with f: For all $J \in \mathscr{J}$ and $\mathbf{R} \in \mathcal{D}^J$, (a) if every voter $i \in N$ is an extreme dichotomist, then $V_f^J(\mathbf{R}, f(J))$ is the first alternative $x_k \in Pl(f(J), \mathbf{R})$ according to the numerical order; and (b) if at least one voter $i \in N$ is not an extreme dichotomist, then $V_f^J(\mathbf{R}, f(J)) = x_k$, where x_k is the first alternative in $OPl(J)$ according to the numerical order. Note that for all $J \in \mathscr{J}$ and $\mathbf{R} \in \mathcal{D}^J$, if every voter is an extreme dichotomist, then we have $OPl(J) = Pl(f(J), \mathbf{R})$.

The voting procedure with f is clearly efficient, non-dictatorial, and anonymous. We now show that it is stable. For all $J \in \mathscr{J}$, $i \in N$, $J_i' \in \mathscr{J}_i$, and

$\mathbf{R} \in \mathcal{D}^J \cap \mathcal{D}^{(J'_i, J_{-i})}$, suppose $V_f^J(\mathbf{R}, f(J'_i, J_{-i})) \in f(J)$. If there exists at least one voter $j \in N$ who is not an extreme dichotomist under J, then $f(J)$ is a singleton. Therefore, the voting procedure with f is stable. Suppose every voter $j \in N$ is an extreme dichotomist under J. If $T_i^J(X) \neq T_i^{(J'_i, J_{-i})}(X)$, then we have $\mathcal{D}^J \cap \mathcal{D}^{(J'_i, J_{-i})} = \emptyset$, which implies that the voting procedure with f is stable. Suppose $T_i^J(X) = T_i^{(J_i, J_{-i})}(X)$. Then, we have $f(J) = f(J'_i, J_{-i})$. Therefore, we have $V_f^J(\mathbf{R}, f(J)) = V_f^{(J'_i, J_{-i})}(\mathbf{R}, f(J'_i, J_{-i}))$ by construction of $\{V_f^J\}$. Thus, the voting procedure with f is stable.

Finally, we show that the voting procedure with f is opinion-based strategy-proof. Let $J \in \mathscr{J}$, $i \in N$, and $\mathbf{R} \in \mathcal{D}^J$. Suppose $V_f^J(\mathbf{R}, f(J)) \in X_-^{J_i}$. Let $x_k \in X_+^{J_i}$. We show that it is impossible to have $x_k = V_f^{(J'_i, J_{-i})}(R'_i, \mathbf{R}_{-i}, f(J'_i, J_{-i}))$ for some $J'_i \in \mathscr{J}_i$ and some $R'_i \in \mathcal{D}_i^{(J'_i, J_{-i})}$. We distinguish two possible cases: (i) there exists at least one voter $j \in N \backslash \{i\}$ who is not an extreme dichotomist under J; and (ii) every voter $j \in N \backslash \{i\}$ is an extreme dichotomist under J.

Case (i). By construction of $\{V_f^J\}$, $f(J)$ is a singleton, and $V_f^J(\mathbf{R}, f(J))$ is only the alternative in $f(J)$. In addition, $V_f^J(\mathbf{R}, f(J))$ is an alternative such that the greatest number of voters have positive opinions about it. Note that because we have $V_f^J(\mathbf{R}, f(J)) \in X_-^{J_i}$, $N_+^J(V_f^J(\mathbf{R}, f(J))) < n$. If we have $\#OPl(J) \geq 2$, $V_f^J(\mathbf{R}, f(J))$ is the first alternative in $OPl(J)$, according to the numerical order.

Because $x_k \in X_+^{J_i}$ and $V_f^J(\mathbf{R}, f(J)) \in X_-^{J_i}$, we have $N_+^{(J'_i, J_{-i})}(x_k) \leq N_+^J(x_k)$ and $N_+^J(V_f^J(\mathbf{R}, f(J))) \leq N_+^{(J'_i, J_{-i})}(V_f^J(\mathbf{R}, f(J)))$. Because $x_k \neq V_f^J(\mathbf{R}, f(J))$, we have either $N_+^J(x_k) < N_+^J(V_f^J(\mathbf{R}, f(J)))$ or $N_+^J(x_k) = N_+^J(V_f^J(\mathbf{R}, f(J))) < n$, and x_k follows numerically after $V_f^J(\mathbf{R}, f(J))$. In the former case, we have $N_+^{(J'_i, J_{-i})}(x_k) < N_+^{(J'_i, J_{-i})}(V_f^J(\mathbf{R}, f(J)))$. We have $x_k \neq V_f^{(J'_i, J_{-i})}(R'_i, \mathbf{R}_{-i}, f(J'_i, J_{-i}))$ by construction of $\{V_f^J\}$. In the latter case, we have $N_+^{(J'_i, J_{-i})}(x_k) < n$, and $N_+^{(J'_i, J_{-i})}(x_k) \leq N_+^{(J'_i, J_{-i})}(V_f^J(\mathbf{R}, f(J)))$ and x_k follows numerically after $V_f^J(\mathbf{R}, f(J))$. We have $x_k \neq V_f^{(J'_i, J_{-i})}(R'_i, \mathbf{R}_{-i}, f(J'_i, J_{-i}))$ by construction of $\{V_f^J\}$.

Case (ii). For all $J'' \in \mathscr{J}$, all $\mathbf{R}'' \in \mathcal{D}^{J''}$, and all $x_l \in X$, if every voter is an extreme dichotomist under J'', then we have $N_+^{J''}(x_l) = N^{\mathbf{R}''}(x_l, f(J''))$, which implies $OPl(J'') = Pl(f(J''), \mathbf{R}'')$. In addition, by construction of $\{V_f^{J''}\}$, $V_f^{J''}(\mathbf{R}'', f(J''))$ is always the first alternative in $Pl(f(J''), \mathbf{R}'')(= OPl(J''))$, according to the numerical order. Therefore, if voter i is an extreme dichotomist under J, then in Case (i), replace $N_+^J(x_k)$ with $N^{\mathbf{R}}(x_k, f(J))$ and $N_+^J(V_f^J(\mathbf{R}, f(J)))$ with $N^{\mathbf{R}}(V_f^J(\mathbf{R}, f(J)), f(J))$. Similarly, if voter i is an extreme dichotomist under (J'_i, J_{-i}), then in Case (i), replace $N_+^{(J'_i, J_{-i})}(x_k)$ with $N^{(R'_i, \mathbf{R}_{-i})}(x_k, f(J'_i, J_{-i}))$ and $N_+^{(J'_i, J_{-i})}(V_f^J(\mathbf{R}, f(J)))$ with $N^{(R'_i, \mathbf{R}_{-i})}(V_f^J(\mathbf{R}, f(J)), f(J'_i, J_{-i}))$. Then, we can show that the voting procedure with f is opinion-based strategy-proof in a similar way to Case (i). $\square$

Erdamar et al. (2017) show that no efficient and anonymous (one-stage) voting procedure exists that satisfies evaluationwise strategy-proofness in the preference-approval model when a preference domain is sufficiently rich. In contrast, we find an efficient and anonymous voting procedure with f that satisfies opinion-based strategy-proofness and stability. Evaluationwise strategy-proofness in the preference-approval model is technically equivalent to opinion-based strategy-proofness in the proposed model. Therefore, Theorem 3 implies that the impossibility in the preference-approval model is resolved by including a nomination process in the voting procedure.

6 Concluding remarks

In this study, we investigated the strategic manipulation of two-stage voting procedures with the nomination process. In the first (nomination) process, some alternatives are nominated by aggregating the voters' opinions, which are positive or negative views about which alternatives are eligible as candidates for collective choice. In the second (voting) process, the voting outcome is selected from the set of nominated alternatives by aggregating the voters' preferences.

The model has a richer structure than that of the preference-approval model, where voters simultaneously express their preferences, which rank alternatives, and their evaluations of whether each alternative is acceptable or unacceptable. The preference-approval model does not include a nomination process because collective choice occurs through a one-stage voting procedure in which an alternative is selected from a fixed set of alternatives.

We extended the notion of strategy-proofness to voting procedures with f, and weakened strategy-proofness to what we call opinion-based strategy-proofness. In addition, we proposed a new notion of non-manipulability for strategic nomination, called stability.

Strategy-proofness in the model drastically restricts both the class of nomination rules f and the class of voting rules with $f(J)$. We first showed that the joint satisfaction of strategy-proofness and stability implies opinion invariance, which requires that voting outcomes be independent of the voters' opinions (Theorem 1). Opinion invariance is a demanding property in the model, because it means that the voting outcomes must always be chosen from the intersection of any two sets of nominated alternatives, which ought to vary with the voters' opinions. Therefore, if the intersection of two sets of nominated alternatives is empty, then every voting procedure with f trivially violates opinion invariance. Thus, opinion invariance is not plausible in the model. Therefore, we have to exclude either strategy-proofness or stability from the list of axioms.

Second, we showed that a necessary and sufficient condition for an efficient, non-dictatorial, and strategy-proof voting procedure with f to exist is that $\#f(J^*) = 2$, where every voter has a positive opinion about each alternative (Theorem 2). As a corollary to Theorem 2, we showed that a Gibbard–Satterthwaite-type impossibility is still valid in the proposed model. This result has a stronger implication than the Gibbard–Satterthwaite theorem in the standard Arrovian social choice model, because if $\#f(J^*) \geq 3$ and a voting procedure with f is efficient and strategy-proof, then there exists a dictator on $f(J)$, for all $J \in \mathscr{J}$. Furthermore, the nomination rule f is under the control of the dictator in that his or her top set of alternatives in $f(J)$ under J must be equivalent to his or her top set of alternatives in $f(J^*)$ under J.

Although Theorems 1 and 2 have negative implications for strategy-proof two-stage voting procedures, we obtain a possibility result if strategy-proofness is weakened to opinion-based strategy-proofness. We showed that there exists an efficient, non-dictatorial, and anonymous two-stage voting procedure that is opinion-based strategy-proof and stable (Theorem 3). This result has an opposite implication to that of the impossibility result of Erdamar et al. (2017), where every efficient and anonymous (one-stage) voting procedure is evaluation-wise manipulable when a preference domain is sufficiently rich.

Finally, as mentioned in Section 3, we employ a reduced form of the Iwata (2016, 2018) two-stage collective choice model. In the latter model, opinions are trichotomous and voting rules with $f(J)$ are set-valued. Thus, a non-trivial question is whether it is possible to extend our results to include the two-stage collective choice model. However, it is not clear how to extend opinion-based strategy-proofness and stability to the model of the Iwata (2016, 2018). The existence of a voting procedure with f that we use to prove Theorem 3 depends on our notion of opinion-based strategy-proofness and stability. Therefore, this problem requires further research.

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