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Fiscal and Monetary Policies in a Keynesian Model of Endogenous Growth Cycle with Public Debt Accumulation

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Abstract

In this paper, we study the dynamic effect of fiscal and monetary policies in a high-dimensional Keynesian model of endogenous growth cycle with public debt accumulation. The reduced form of our model is described by a five-dimensional system of nonlinear differential equations. The dynamic effect of the fiscal and monetary policy mix on the macroeconomic stability, instability and cyclical fluctuations are studied both analytically and numerically. We consider what policy mix is appropriate or inappropriate from the common sense standard of policy evaluation.

Key words : Keynesian model of endogenous growth cycle, Public debt accumulation, Budget constraint of consolidated government, Fiscal and monetary policy mix, Inflation targeting, Credibility

JEL codes : E12, E31, E32, E62, E63

1. Introduction

In this paper, we study the impact of the fiscal and monetary policy mix on macroeconomic stability by using a variant of the 'high-dimensional Keynesian macrodynamic model' that was developed by Asada, Chiarella, Flaschel and Franke(2003, 2010). [1] This paper is a sequel to a series of the related works such as Asada(2006a, 2006b, 2010, 2011) and Asada and Ouchi(2009).

In particular, the model in this paper is an integration of the models that were presented in Asada(2010, 2011) and Asada and Ouchi(2009). Asada(2010) studied the effect of the 'Taylor rule' type interest rate monetary policy by using a relatively small scale dynamic model that is described by means of a two-dimensional system of differential equations. In Asada(2010), the dimension of the system could be kept low enough because both of the dynamic effect of capital accumulation and that of public debt accumulation were ignored. Asada(2011) presented a four-dimensional extended version of the model by introducing the effect of public debt accumulation into the model. Even in this model, however, the capital accumulation effect was ignored. On the other hand, Asada and Ouchi(2009) formulated a four-dimensional model of 'Keynesian endogenous growth cycle' by introducing Kaldorian type technical progress function and capital accumulation effect. The spirit of this model is 'Keynesian' in the sense that both of the under-employment of labor and under-utilization of capital stock due to the insufficient effective demand are allowed for unlike 'neoclassical' endogenous growth model represented by Barro and Sara-i-Martin(2004). [2] In Asada and Ouchi(2009), however, the Taylor rule type interest rate monetary policy was not studied. In this paper, we present an integrated five-dimensional dynamic model that considers both of public debt accumulation effect and capital accumulation effect explicitly, and investigate the dynamic effect of the fiscal and monetary policy mix on the macroeconomic stability, instability and cyclical fluctuations both analytically and numerically. We investigate what is the appropriate policy mix to 'stabilize an unstable economy'.[3]

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[1] 'High-dimensional' dynamic model implies the dynamic model with many (in fact, more than three) variables. 'Dimension' of the system is equal to the number of the variables.

[2] Asada and Ouchi(2009) provide a theoretical counterargument to Hayashi and Prescott(2002) who assert on the basis of the 'real business cycle' theory that the cause of Japanese deflationary depression is not the insufficient effective demand but the 'technological shock'. In Asada and Ouchi(2009)'s model, like Kaldor(1957), the rate of technical progress per se is determined by the effective demand through investment expenditure.

[3] Obviously, this phrase is a quotation from the title of Minky's famous book (Minsky 1982). See also Asada, Chiarella, Flaschel, Mouakil, Proaño and Semmler(2010).

At first glance our model is somewhat similar to the now fashionable 'New Keynesian' dynamic model represented by Woodford(2003) and Galí(2008). In fact, core parts of both models are some types of 'IS curve', 'Phillips curve', and Taylor type interest rate monetary policy rule. It is important to note, however, that there are some critical differences between these two models. Typical 'New Keynesian' dynamic model is based on the premise that all economic agents including policy makers can behave perfectly rationally by solving the complicated dynamic optimization problems in the environment of perfect foresight or rational expectations. As Mankiw(2001) correctly pointed out, however, the 'New Keynesian Phillips curve' that is based on such a premise produces empirically implausible paradoxical behavior.[4] Moreover, 'New Keynesian' rational expectations approach treats the variables such as actual and expected rates of inflation and nominal rate of interest as 'jump variables' that transforms the unstable equilibrium point (in the traditional sense) to the 'stable' equilibrium point. But, as Mankiw(2001) noted, such a premise contradicts the fact that the effect of monetary policy on inflation is 'delayed and gradual'. On the other hand, high-dimensional dynamic Keynesian approach by Asada, Chiarella, Flaschel and Franke(2003, 2010), on which our model is based, is immune from such paradoxes because it is based on more traditional approach, so to speak, 'old Keynesian' approach in the sense of Tobin(1994).

In this 'old Keynesian' approach, there is no 'jump variable', because the initial conditions of all variables are given historically, and the economic agents including the policy makers act 'bounded rationally' in the sense of Akerlof and Shiller(2009) in the world with imperfect knowledge and imperfect information, although it does not necessarily mean that their behaviors are irrational. In this paper, we do not adopt the usual evaluation criterion whether a policy mix is 'optimal' or not. Instead, we adopt the criterion whether it is 'appropriate' or not from the common sense standard of policy evaluation. In fact, we consider that the fiscal and monetary policy mix that can *stabilize* the macroeconomic system is *appropriate* (or permissible), while a policy mix that has a *destabilizing* effect is considered to be *inappropriate* (or impermissible). In our model, none of the economic agents including the policy makers have the ability to calculate Hamiltonian under perfect foresight or rational expectation. Nevertheless, their behaviors can be 'appropriate' rather than 'optimal' from the common sense standard of evaluation in some situations.

[4] Mankiw(2001) pointed out that the price inflation rate *accelerates* whenever the current output level is *below* the natural output level. This is called the New Keynesian 'sign reversal' problem (see Asada 2010 and Asada, Chiarella, Flaschel and Franke 2010).

2. Formulation of the model

Our model consists of the following system of equations, where a dot over a symbol denotes the derivative with respect to time.[5]

$$y = c + i + g \;;\; y = Y/K,\quad c = C/K,\quad i = I/K,\quad g = G/K \tag{1}$$

$$c = \delta(y + rb - t) + c_0 \;;\; 0 < \delta < 1,\; c_0 > 0,\; b = B/(pK),\quad t = T/K \tag{2}$$

$$t = \tau(y + rb) - t_0 \;;\; 0 < \tau < 1,\; t_0 > 0 \tag{3}$$

$$i = i(r - \pi^e) \;;\; i_{r-\pi^e} = \partial i / \partial(r - \pi^e) < 0 \tag{4}$$

$$m(r)h = \phi(r)y \;;\; m_r = dm/dr > 0,\; \phi_r = d\phi/dr < 0,\; h = H/(pK) \tag{5}$$

$$T + \dot{B}/p + \dot{H}/p = G + rB/p \tag{6}$$

$$\dot{w}/w = \kappa(e - \bar{e}) + \dot{a}/a + \pi^e \;;\; \kappa > 0,\, 0 < e = N/N^s \leqq 1 \tag{7}$$

$$p = (1+v)wN/Y = (1+v)w/a \;;\; v > 0 \tag{8}$$

$$\pi = \dot{p}/p \tag{9}$$

$$\dot{e}/e = \dot{y}/y + \dot{K}/K - \dot{a}/a - n_s \;;\; n_s = \dot{N}^s/N^s > 0 \tag{10}$$

$$\dot{K} = I + \sigma G \;;\; 0 < \sigma < 1 \tag{11}$$

$$\dot{a}/a = \varepsilon(\dot{K}/K) + \varepsilon_0 \;;\; 0 < \varepsilon < 1,\quad \varepsilon_0 > 0 \tag{12}$$

$$\dot{g} = \alpha\{\theta(\bar{e} - e) + (1-\theta)(\bar{b} - b)\} \;;\; \alpha > 0,\, 0 < \theta < 1 \tag{13}$$

$$\dot{r} = \begin{cases} \beta_1(\pi - \bar{\pi}) + \beta_2(e - \bar{e}) & if \quad r > 0 \\ \max[0, \beta_1(\pi - \bar{\pi}) + \beta_2(e - \bar{e})] & if \quad r = 0 \end{cases} \;;\; \beta_1 > 0,\; \beta_2 > 0 \tag{14}$$

$$\dot{\pi}^e = \gamma[\xi(\bar{\pi} - \pi^e) + (1-\xi)(\pi - \pi^e)] \;;\; \gamma > 0,\, 0 \leqq \xi \leqq 1 \tag{15}$$

The meanings of the symbols of the endogenous variables are as follows. Y = real national income (real output). C = real private consumption expenditure. I = real private investment expenditure. G = real government expenditure. K = real capital stock. y = output-capital ratio, which is proportional to the rate of capacity utilization of capital stock. c = private consumption-capital ratio. i = private investment-capital ratio (rate of private investment). g = government expenditure-capital ratio (rate of government expenditure). T = real income tax. B = nominal public debt. t = income tax-capital ratio. b = public debt-capital ratio. p = price level. r = nominal interest rate of public debt. H = nominal high-powered money (nominal base money). h = high-powered money-capital ratio. w = nominal wage rate. π = rate of price inflation. π^e = expected rate of price inflation. e = rate of employment = 1 − rate of unemployment. N = labor employment. N^s = labor supply. $a = Y/N$ = average labor

[5] This system of equations is in fact a synthesis of the system of equations in Asada and Ouchi(2009) and that in Asada(2011).

productivity.

On the other hand, δ, c_0, τ, t_0, κ, v, n_s, σ, ε, ε_0, α, θ, β_1, β_2, γ, ξ, $\bar{e}$, $\bar{b}$ and $\bar{\pi}$ are parameters, where we assume that $0 < \bar{e} < 1$, $\bar{b} > 0$ and $\bar{\pi} > 0$.

Next, let us explain how these equations are derived. Eq. (1) is the equilibrium condition for the goods market. [6] We assume that the output-capital ratio at the full capital utilization (y_f) is a positive constant. Nevertheless, the actual output-capital ratio (y) becomes a variable rather than a constant through the relationship

$$y = uy_f \quad ; 0 \leqq u \leqq 1, \tag{16}$$

where u is the rate of capital utilization.

Eq. (2) is the Keynesian consumption function (see Keynes 1936), where the consumption out of the interest of the public debt is explicitly considered. Eq. (3) is the income tax function. Eq. (4) is the standard Keynesian investment function, which implies that the private investment rate is a decreasing function of the 'expected real interest rate' $(r - \pi^e)$.[7]

Eq. (5) is the equilibrium condition for the money market (LM equation). The right hand side of this equation is the Keynesian real money demand per capital stock, and the left hand side is the real money supply per capital stock, where $m(r) > 1$ is the money multiplier. In other words, the nominal money supply (M) can be written as

$$M = m(r)H. \tag{17}$$

Eq. (6) is the budget constraint of the 'consolidated government' that includes the central bank. The right hand side of this equation is the real government expenditure including the interest payment of the public debt, and the left hand side is its resource. In other words, the government expenditure including the interest payment of the public debt must be financed by either of taxation, debt financing or money financing.[8]

Eq. (7) is a standard version of the 'expectations-augmented wage Phillips curve',

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[6] We neglect the international trade and the international capital movement for simplicity.

[7] Needless to say, the entrepreneurs' investment activities will be affected by the subjective factors that are conveniently summarized by the term 'animal spirits' apart from the expected real rate of interest (see Keynes 1936 and Akerlof and Shiller 2009). In this paper, however, we do not consider this subtle problem. Incidentally, Asada, Chiarella, Flaschel and Franke(2010) Chap. 6 treat such a subjective factor that drives the economy formally by using the concept of the 'investment climate'.

[8] This formulation of the budget constraint of the 'consolidated government' is due to Turnovsky(2000) Part I.

which describes the dynamic of the labor market that reflects the wage bargaining between capitalists and workers. Eq. (8) is a formalization of the mark up pricing behavior of the imperfectly competitive firms, where $(1+v)$ is the mark up over prime cost that reflects the 'degree of monopoly' of the economy (see Kalecki 1971). Eq. (9) is simply the definition of the rate of price inflation. Differentiating Eq. (8) and considering Eq. (9), we have

$$\pi = \dot{p}/p = \dot{w}/w - \dot{a}/a. \tag{18}$$

Substituting Eq. (7) into Eq. (18), we obtain the following 'expectations-augmented price Phillips curve'.

$$\pi = \kappa(e-\bar{e}) + \pi^e \tag{19}$$

We can derive Eq. (10) as follows. By definition, we have

$$e = N/N^s = \frac{(Y/K)K}{(Y/N)N^s} = \frac{yK}{aN^s}. \tag{20}$$

Differentiating this equation with respect to time, we obtain Eq. (10).

Eq. (11) means that all of the private investment expenditure and a fraction σ of the government expenditure contribute to the increase of the capital stock (capital accumulation).[9] Substituting Eq. (4) into this equation, we have the following expression of the rate of capital accumulation $(\dot{K}/K)$.

$$\dot{K}/K = I/K + \sigma G/K = i(r-\pi^e) + \sigma g \tag{21}$$

Eq. (12) is a version of the 'Kaldorian' technical progress function, which implies that the rate of technical progress $(\dot{a}/a)$ is positively correlated with the rate of capital accumulation (see Kaldor 1957). Substituting Eq. (21) into this equation, we obtain

$$\dot{a}/a = \varepsilon[i(r-\pi^e)+\sigma g] + \varepsilon_0 = \varphi(r-\pi^e, g). \tag{22}$$

This equation implies that the rate of technical progress is endogenously determined by several economic variables in this model. In this case, the 'natural rate of growth' (n) as well as the actual rate of growth is not constant any longer, but it becomes an endogenous variable that is determined by the following formula.

$$n = n^s + \dot{a}/a = n^s + \varphi(r-\pi^e, g) = n(r-\pi^e, g) \tag{23}$$

This means that our model is a Keynesian (or Kaldorian) model of endogenous growth in contrast to the neoclassical model of endogenous growth that is represented by Barro and Sala-i-Martin (2004).

Eq. (13) is a formalization of the fiscal policy rule that considers both of employment and balance of public debt, where $\bar{e}$ is the 'natural' rate of employment that is consistent with Eq. (7), and $\bar{b}$ is the target value of the public debt-capital ratio that is

[9] We neglect the capital depreciation for simplicity.

set by the government. The parameter θ is the weight of the importance of the employment consideration that is determined by the government.[10]

Eq. (14) is a formalization of the monetary policy rule in spirit of the 'Taylor rule' due to Taylor(1993) (see Asada 2010, 2011). In this formulation, the 'nonnegative' constraint, which means that the nominal rate of interest cannot become negative, is considered. The parameter $\bar{\pi}$ is the target rate of inflation that is set by the central bank. This monetary policy rule is the interest rate rule that is called the 'flexible inflation targeting', which is the mixture of the inflation targeting and the employment targeting.[11]

Eq. (15) is a formalization of the inflation expectation formation by the public. This is a mixture of the 'forward looking' and 'backward looking'(or 'adaptive) expectations. The parameter ξ is the weight of the 'forward looking' expectation formation, which can be interpreted as the measure of the 'credibility' of the central bank's inflation targeting. We can consider that the higher the value of the parameter ξ, the more credible will be the central bank's announcement concerning the inflation targeting.

A system of equations (1) – (15) is enough to determine the dynamics of 24 endogenous variables Y, C, I, G, K, y, c, i, g, T, B, t, b, p, r, H, h, w, π, π^e, e, N, N^s and a.[12]

3. Derivation of a system of fundamental dynamic equations

Next, we shall transform the extensive system of equations (1) – (15) into the more compact reduced form. First, substituting equations (2) – (4) into Eq. (1), we have the following 'IS equation' that describes the equilibrium condition for the goods market, which determines the rate of capacity utilization of the capital stock (u) through Eq. (16).

$$y = \frac{1}{1-\delta(1-\tau)}\{\delta(1-\tau)rb + \delta t_0 + c_0 + i(r-\pi^e) + g\} = y(r, \pi^e, b, g)$$

$$;\quad y_r = \partial y / \partial r = \{\delta(1-\tau)b + \underset{(-)}{i_{r-\pi^e}}\} / \{1-\delta(1-\tau)\},$$

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[10] This formulation is an adaptation from Asada(2011).

[11] For the empirical and theoretical analyses of inflation targeting, see Krugman(1998), Barnanke, Laubach, Mishkin and Posen(1999), Woodford(2003), Barnanke and Woodford (eds.)(2005), Galí(2008), Asada(2006a, 2006b, 2010, 2011), and Asada and Ouchi(2009).

[12] Eq. (1) contains 5 equations, Eq. (2) contains 3 equations, and each of equations (5), (7) and (10) contains 2 equations respectively. Therefore, a system of equations (1) – (15) in fact contains 24 independent equations including some definitional equations.

$$y_{\pi^e} = \partial y / \partial \pi^e = -\underset{(-)}{i_{r-\pi^e}} / \{1-\delta(1-\tau)\} > 0,$$

$$y_b = \partial y / \partial b = \delta(1-\tau) r / \{1-\delta(1-\tau)\} \gtreqless 0,$$

$$y_g = \partial y / \partial g = 1/\{1-\delta(1-\tau)\} > 1 > 0 \tag{24}$$

Next, we can rewrite Eq. (5) (LM equation) as follows.

$$h = H/(pK) = \psi(r) y \;\; ; \;\; \psi(r) = \phi(r)/m(r), \quad \psi'(r) < 0 \tag{25}$$

Substituting Eq. (24) into Eq. (25), we obtain

$$h = \psi(r) y(r, \pi^e, b, g) = h(r, \pi^e, b, g). \tag{26}$$

Differentiating the definitional equation $h = H/(pK)$ with respect to time and substituting capital accumulation equation (Eq. (21)) into it, we have

$$\dot{H}/(pK) = (\pi + \dot{K}/K) h + \dot{h} = \{\pi + i(i-\pi^e) + \sigma g\} h + \dot{h}. \tag{27}$$

Differentiating Eq. (26) with respect to time and substituting it into Eq. (27), we obtain the following expression.

$$\begin{aligned} \dot{H}/(pK) &= \{\pi + i(r-\pi^e) + \sigma g\} h + \psi'(r) y \dot{r} + \psi(r) \dot{y} \\ &= \{\pi + i(r-\pi^e) + \sigma g\} h(r, \pi^e, b, g) + \{\psi'(r) y + \psi(r) y_r\} \dot{r} \\ &\quad + \psi(r) \{y_{\pi^e} \dot{\pi}^e + y_b \dot{b} + y_g \dot{g}\} \end{aligned} \tag{28}$$

Equations (26) and (28) mean that the time path of the high-powered money (H) is determined endogenously when the central bank selects the nominal rate of interest rather than the high-powered money as a policy variable.

Next, differentiating another definitional equation $b = B/(pK)$ with respect to time and substituting Eq. (21) into it, we have

$$\dot{B}/(pK) = (\pi + \dot{K}/K) b + \dot{b} = \{\pi + i(r-\pi^e) + \sigma g\} b + \dot{b}. \tag{29}$$

On the other hand, it follows from Eq. (6) that

$$\dot{B}/(pK) + \dot{H}/(pK) = g + rb - t. \tag{30}$$

Substituting equations (3), (24), (28), and (29) into Eq. (30), we obtain the following expression.

$$\begin{aligned} \dot{b} = [&g - \tau y(r, \pi^e, b, g) + t_0 + \{r(1-\tau) - \pi - i(r-\pi^e) - \sigma g\} b \\ &- \{\pi + i(r-\pi^e) + \sigma g\} h(r, \pi^e, b, g) - \{\psi'(r) y(r, \pi^e, b, g) + \psi(r) y_r\} \dot{r} \\ &- \psi(r) \{y_{\pi^e} \dot{\pi}^e + y_g \dot{g}\}] / \{1 + \psi(r) y_b\} \end{aligned} \tag{31}$$

Finally, we have the following five dimensional system of nonlinear differential equations substituting (19), (21), and (22) into equations (10), (13), (14) and (15), and furthermore substituting these relationships into Eq. (31).

(i) $\dot{g} = F_1(b,e) = \alpha\{\theta(\bar{e}-e) + (1-\theta)(\bar{b}-b)\}$

(ii) $\dot{r} = F_2(\pi^e, e) = \begin{cases} (\kappa\beta_1 + \beta_2)(e-\bar{e}) + \beta_1(\pi^e - \bar{\pi}) & if \quad r > 0 \\ \max[0, (\kappa\beta_1 + \beta_2)(e-\bar{e}) + \beta_1(\pi^e - \bar{\pi})] & if \quad r = 0 \end{cases}$

(iii) $\dot{\pi}^e = F_3(\pi^e, e) = \gamma[\xi(\bar{\pi} - \pi^e) + (1-\xi)\kappa(e-\bar{e})]$

(iv) $\dot{b} = F_4(g, r, \pi^e, b, e) = [g - \tau y(r, \pi^e, b, g) + t_0 + \{r(1-\tau) - \kappa(e-\bar{e}) - \pi^e$

$- i(r-\pi^e) - \sigma g\}b - \{\pi + i(r-\pi^e) + \sigma g\}h(r, \pi^e, b, g)$
$- \{\psi'(r)y(r, \pi^e, b, g) + \psi(r)y_r\}F_2(\pi^e, e)$

$- \psi(r)\{y_{\pi^e}F_3(\pi^e, e) + y_g F_1(b,e)\}]/\{1 + \psi(r)y_b\}$

(v) $\dot{e} = F_5(g, r, \pi^e, b, e) = e[\{y_r F_2(\pi^e, e) + y_{\pi^e}F_3(\pi^e, e) + y_b F_4(g, r, \pi^e, b, e)$

$+ y_g F_1(b,e)\}/y(r, \pi^e, b, g) + (1-\varepsilon)\{i(r-\pi^e) + \sigma g\} - (\varepsilon_0 + n_s)]$ (32)

This system may be called a system of 'fundamental dynamic equations' in our model.

4. Characteristics of the long run equilibrium solution

In this section, we shall consider the 'long run equilibrium' solution of the system (32) that satisfies

$$\dot{g} = \dot{r} = \dot{\pi}^e = \dot{b} = \dot{e} = 0, \quad e = \bar{e}. \tag{33}$$

Substituting these conditions into Eq. (32), we have the following set of conditions for the long run equilibrium values $(g^*, r^*, \pi^{e*}, b^*, e^*)$.

(i) $e^* = \bar{e}, \quad b^* = \bar{b}, \quad \pi^{e*} = \pi^* = \bar{\pi}$

(ii) $i(r^* - \bar{\pi}) = \dfrac{\varepsilon_0 + n_s}{1-\varepsilon} - \sigma g^*$

(iii) $g^* - \tau y(r^*, \bar{\pi}, \bar{b}, g^*) + t_0 + \{r^*(1-\tau) - \bar{\pi} - \dfrac{\varepsilon_0 + n_s}{1-\varepsilon}\}\bar{b}$

$- \{\bar{\pi} + \dfrac{\varepsilon_0 + n_s}{1-\varepsilon}\}h(r^*, \bar{\pi}, \bar{b}, g^*) = 0$ (34)

Eq. (34) means that the 'natural' rate of employment, the target rate of public debt-capital ratio, and the target rate of price inflation are realized at the long run equilibrium point. We can determine the equilibrium values of the government expenditure-capitsal ratio (g^*) and the nominal rate of interest of the public debt (r^*) by solving a set of simultaneous equations (34) (ii) and (iii). We can see that r^* cannot be positive if the target rate of inflation $(\bar{\pi})$ is too low(especially non-positive). In such a case, the economically meaningful long run equilibrium does not exist, because the nominal interest rate cannot be negative. This means that the sufficiently large value of target rate of inflation (for example, two or three percent per year) is required to ensure the existence of economically meaningful long run equilibrium. In this paper, we *assume* that this system of equations has the unique economically meaningful solution $(g^*, r^*) > (0,0)$.[13]

Substituting Eq. (34) (ii) into equations (21), (22) and (23), we obtain the following relationships at the long run equilibrium point.

$$(\dot{K}/K)^* = \frac{\varepsilon_0 + n_s}{1-\varepsilon} > 0 \tag{35}$$

$$(\dot{a}/a)^* = \frac{\varepsilon_0 + \varepsilon n_s}{1-\varepsilon} > 0 \tag{36}$$

$$n^* = n_s + (\dot{a}/a)^* = \frac{\varepsilon_0 + n_s}{1-\varepsilon} = (\dot{K}/K)^* > 0 \tag{37}$$

Eq. (35) gives the equilibrium rate of capital accumulation. Eq. (36) gives the equilibrium rate of technical progress. Eq. (37) means that the equilibrium value of the 'natural' rate of growth is equal to the equilibrium rate of capital accumulation.

It follows from equations (17) and (26) that we have

$$M^* = m(r^*)H^*, \tag{38}$$

$$h^* = (H/pK)^* = h(r^*, \bar{\pi}, \bar{b}, g^*) = \text{constant} \tag{39}$$

at the long run equilibrium point, which implies that we have

$$(\dot{M}/M)^* = (\dot{H}/H)^* = \pi^* + (\dot{K}/K)^* = \bar{\pi} + \frac{\varepsilon_0 + n_s}{1-\varepsilon} > 0 \tag{40}$$

at the long run equilibrium point, which gives the equilibrium growth rate of money supply and that of high-powered money.

5. Analysis of local stability/instability and cyclical fluctuations

[13] It is worth noting that all of the the long run equilibrium values of the economic variables are *independent of* fiscal policy parameters (α and θ), monetary policy parameters (β_1 and β_2), and parameters of the inflation expectation formation equation (γ and ξ).

Next, we shall study the local stability/instability of the long run equilibrium point. For this purpose, let us consider the following (5×5) Jacobian matrix of the system (32) that is evaluated *at the equilibrium point.*

$$J = \begin{bmatrix} 0 & 0 & 0 & -\alpha(1-\theta) & -\alpha\theta \\ 0 & 0 & \beta_1 & 0 & \kappa\beta_1 + \beta_2 \\ 0 & 0 & -\gamma\xi & 0 & \gamma(1-\xi)\kappa \\ F_{41} & F_{42} & F_{43} & F_{44} & F_{45} \\ F_{51} & F_{52} & F_{53} & F_{54} & F_{55} \end{bmatrix} \tag{41}$$

The detailed expressions of the partial derivatives F_{ij} are given in **Appendix A**.

The characteristic equation of this system becomes

$$\Gamma(\lambda) = |\lambda I - J| = \lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0, \tag{42}$$

where

$$a_1 = -traceJ = \gamma\xi - F_{44} - F_{55}, \tag{43}$$

$$a_j = (-1)^j \text{(sum of all principal j-th order minors of } J \text{)} \quad (j = 2,3,4), \tag{44}$$

$$a_5 = -\det J. \tag{45}$$

It is worth noting that the Liénard-Chipart expression of the Routh-Hurwitz conditions for stable roots implies that a set of *necessary* (but not sufficient) conditions for the local stability of the equilibrium point of the system (32) is expressed by

$$a_j > 0 \quad \text{for all} \quad j \in \{1,2,\cdots,5\}.\text{[14]} \tag{46}$$

This means that the equilibrium point of this system is locally unstable if we have $a_j <$ 0 for at least one of $j \in \{1,2,\cdots,5\}$. The following proposition follows from this fact.

Proposition 1.

Suppose that the following set of conditions is satisfied.

(1) Fiscal policy parameter θ is close to 0.

(2) Fiscal policy parameter α is sufficiently large.

(3) Monetary policy parameters β_1 and β_2 are close to 0.

(4) Credibility parameter of the central bank's inflation targeting ξ is close to 0.

Then, the equilibrium point of the system (32) is *locally unstable.*

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[14] See Gandolfo(2009) Chap. 16.

Proof. See **Appendix B**.

The economic interpretation of this proposition will be presented in section 7.

At this stage of the analysis, let us introduce the following assumption.[15]

Assumption 1.

(i) $\left| i_{r-\pi^e} * \right| > \delta(1-\tau)\bar{b}.$

(ii) $\dfrac{(1-\tau)(1-\delta)}{1-\delta(1-\tau)} > \sigma(\bar{b}+h^*).$

(iii) $\tau \underset{(+)}{y_b} * + \{\bar{\pi} + \dfrac{\varepsilon_0 + n_s}{1-\varepsilon}\}\{1+\psi(r^*)\underset{(+)}{y_b} *\} > r^*(1-\tau).$

Assumption 1 (i) means that $y_r^* = (\partial y/\partial r)^* < 0,$ and in this case we have $F_{42} = (\partial \dot{b}/\partial r)^* > 0$ (see equations (24) and (A2)). **Assumption 1** (ii) means that $F_{41} = (\partial \dot{b}/\partial g)^* > 0$ (see Eq. (A1)). These inequalities are quite plausible from empirical point of view. On the other hand, under **Assumption 1** (iii) we have $F_{44} = (\partial \dot{b}/\partial b)^* < 0$ *if the fiscal policy parameter* θ *is close to 1.*

Assumption 1 (i) implies the inequality $F_{42} > 0$, but it does not determine the sign of F_{52}. In this paper, however, we *assume* that the inequality $F_{52} = (\partial \dot{e}/\partial r)^* < 0$ is in fact satisfied. This means that we posit the following additional assumption (see Eq. (A7)).

Assumption 2.

$\underset{(+)}{y_b} * \underset{(+)}{F_{42}} < (1-\varepsilon)\left| i_{r-\pi^e} * \right| y^*.$

It is easy to see that F_{55} becomes a linear decreasing function of the parameters β_1 and β_2 if the value of y_b is sufficiently small.

---

[15] As already noted in **Appendix A**, asterisk(*) means that the values are evaluated *at the equilibrium point.*

Assumption 3.

The value of y_b is so small that we have $\partial F_{55} / \partial \beta_1 > 0$ and $\partial F_{55} / \partial \beta_2 > 0$.

Proposition 2.

In addition to Assumptions 1, 2 and 3, let us assume that the following set of conditions is satisfied.

(1) Fiscal policy parameter θ is less than 1, but it is close to 1.

(2) Fiscal policy parameter α is fixed at any positive value.

(3) Either of the monetary policy parameters β_1 or β_2 is sufficiently large.

(4) Credibility parameter of the central bank's inflation targeting ξ is close to 1 (including the case of $\xi = 1$).

Then, the equilibrium point of the system (32) is *locally asymptotically stable*.

Proof. See **Appendix C**.

Propositions 1 and 2 suggest the following results.

(1) The increase of the fiscal policy parameter θ tends to *stabilize* the system.

(2) The increases of the monetary policy parameters β_1 and β_2 tend to *stabilize* the system.

(3) The increase of the credibility parameter ξ tends to *stabilize* the system.

Therefore, it is likely that the increase of one of such parameters will change the unstable system into the stable system. In this case, there exist at least one 'bifurcation point' of such a parameter value, at which the switching from the unstable region to the stable region occurs.[16] At such a bifurcation point, the characteristic equation (42) must have at least one root with zero real part.

Incidentally, we can prove that the coefficient a_5 that is defined by Eq. (45) is always positive as long as $0 \leqq \theta < 1$, $\beta_1 > 0$, $\beta_2 > 0$, and $0 \leqq \xi \leqq 1$ (for the proof, see **Appendix D**). Therefore, we have

$$\Gamma(0) = a_5 > 0, \tag{47}$$

[16] Needless to say, in some cases there may not exist such a bifurcation point, because in some cases the system may be stable (or unstable) in all relevant range of a parameter depending on the constellation of other parameter values. In principle, it is also possible that the 're-switching' of the instability and stability regions occurs even if the increase of a parameter value changes the unstable system into the stable system ultimately.

which means that the characteristic equation (42) does *not* have the real root such as $\lambda = 0$. It follows from this fact that the characteristic equation (42) has a pair of pure imaginary roots at the 'bifurcation point'.[17] In this case, the *cyclical fluctuations* around the equilibrium point occur at some range of parameter values that are near from the bifurcation point, because of the existence of a pair of complex roots.

6. Numerical simulations

Next, we shall provide some numerical simulations that support the analytical results of the previous sections. Let us assume the following parameter values, functional forms and initial values.[18]

(1) Fixed parameters

$\kappa = 0.8$, $\sigma = 0.6$, $\gamma = 0.9$, $\varepsilon = 0.3$, $n_s = 1\%$ (annual growth rate), $\bar{e} = 0.95$,

$\bar{b} = 0.1$, $\bar{\pi} = 0.2\%$ per period.

(2) Functional forms

Consumption function: $c = \delta(y + rb - t) + c_0 = 0.6(y + rb - t) + 0.08$,

Income tax function: $t = \tau(y + rb) - t_0 = 0.2(y + rb) - 0.5$,

Investment function: $i = a_1(r - \pi^e) + i_0 = 0.8(r - \pi^e) + 1.5$,

LM equation: $h = \psi(r)y = \dfrac{a_2}{a_3 r + a_4} y = \dfrac{0.05}{0.8r + 1} y$,

Technical progress function: $\dot{a} / a = \varepsilon(\dot{K} / K) + \varepsilon_0 = 0.3(\dot{K} / K) + 0.5$.

(3) Initial values

$g(0) \approx 1.51$, $r(0) \approx 0.541\%$ per period, $\pi^e(0) \approx 0.15\%$ per period, $b(0) = 0.15$,

---

[17] This means that the type of bifurcation is 'Hopf bifurcation' type in our model (cf. Gandolfo 2009 chap. 25 and mathematical appendices of Asada, Chiarella, Flaschel and Franke 2003, 2010).

[18] In this simulation, we introduce the exogenous constraint $0.4 \leqq e \leqq 1$, and we assume that the unit time period is 0.1 year. In other words, $t = 100$ means 10 years. The values π, π^e, and r denote per cent per unit time period. For example, $\bar{\pi} = 0.2$ and $r^* = 0.454$ mean that the target rate of inflation is 2% per year and the equilibrium nominal interest rate is 4.54% per year.

$e(0) = 0.9.$

We have the equilibrium values $g^* \approx 0.41$ and $r^* \approx 0.454\%$ per period. Furthermore, we select four parameters ξ, θ, β_1 and β_2 as bifurcation parameters.

Figure 1(Case A) describes the unstable case in which the parameter values ξ, θ, β_1 and β_2 are sufficiently small ($\xi = 0.4,\ \ \theta = 0.3,\ \ \beta_1 = \beta_2 = 0.04$). In this case, the equilibrium point becomes strongly unstable and the 'deflationary depression with the liquidity trap' emerges. It is worth noting that the real interest rate becomes considerably high because of the deflation even if the nominal interest rate is fallen to its lower bound, and the public debt-capital ratio (b) continues to rise in the process of the deflationary depression. In this case, the nominal interest rate is forced to fall to its lower bound not because of the active monetary policy but because of the inactive monetary policy of the central bank.

Figure 2(Case B) describes the case of limit cycles in which the above parameter values are intermediate values ($\xi = \theta = 0.5,\ \ \beta_1 = \beta_2 = 0.05$). Figure 3(Case C) is the stable case in which the above parameter values are sufficiently large ($\xi = \theta = 0.8,\ \ \beta_1 = \beta_2 = 0.1$). These numerical examples support our analytical results.

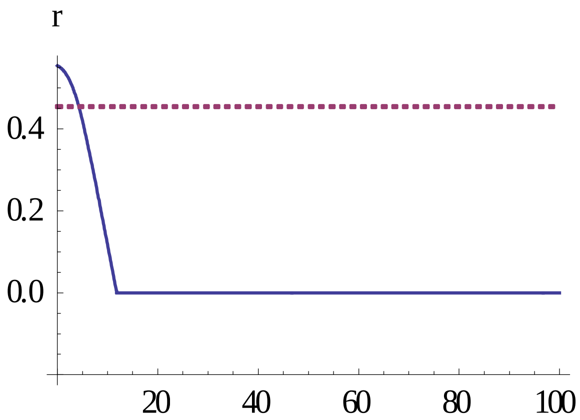
7. Economic interpretation of the analytical results

In this section, we shall try to provide an economic interpretation of the analytical results in this paper.

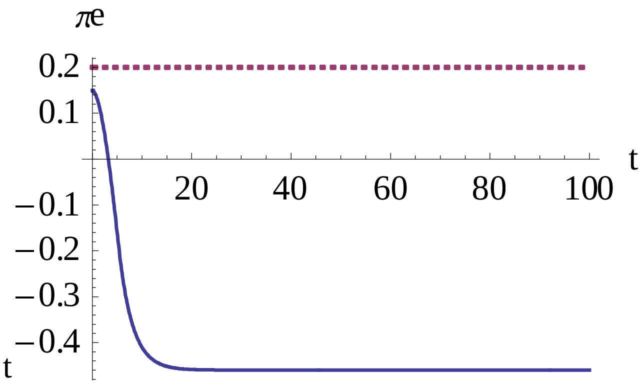
Proposition 1 characterizes a typical inappropriate policy mix. This proposition says that the macroeconomic system tends to be dynamically *unstable* if (1) the government expenditure responds sensitively to the changes of public debt rather than the changes of employment, (2) the central bank's monetary policy is relatively inactive, and (3) the central bank's inflation targeting is relatively incredible so that the public form their inflation expectation rather adaptively (in a backward looking way).[19]

On the other hand, **Proposition 2** characterizes a typical appropriate policy mix. This proposition says that the macroeconomic system tends to be dynamically *stable* if (1) the government expenditure responds sensitively to the changes of employment rather than the changes of public debt, (2) the central bank's monetary policy is relatively active, and (3) the central bank's inflation targeting is credible so that the public can form their

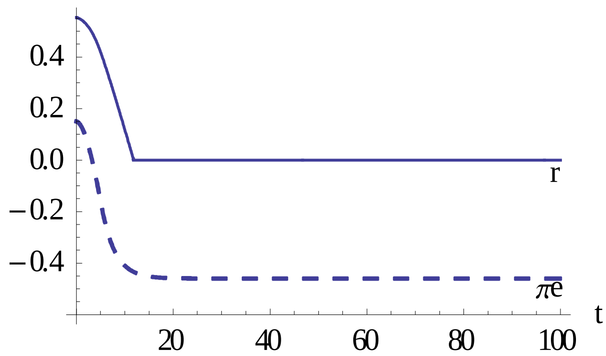
[19] Surprisingly enough, however, this inappropriate policy mix is often adopted by the policy makers, especially in the period of 'deflationary depression' in Japan during such a long period of 20 years (the 1990s and the 2000s), which is called 'lost twenty years' (see Asada2011).



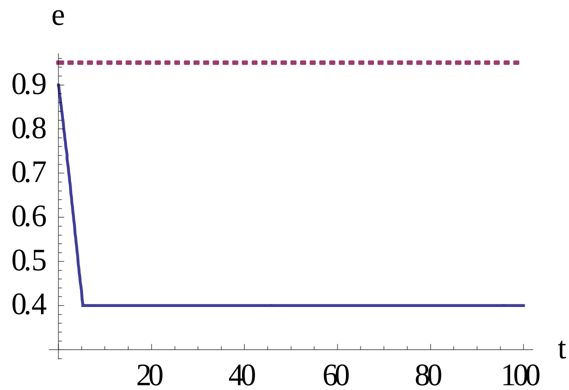
(a) r



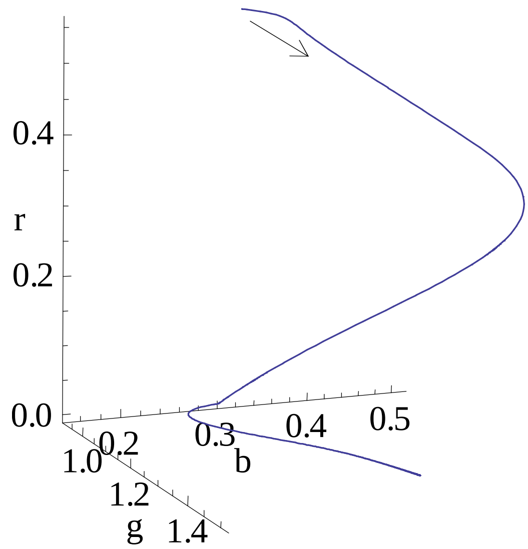
(b) π^e



(a) and (b)

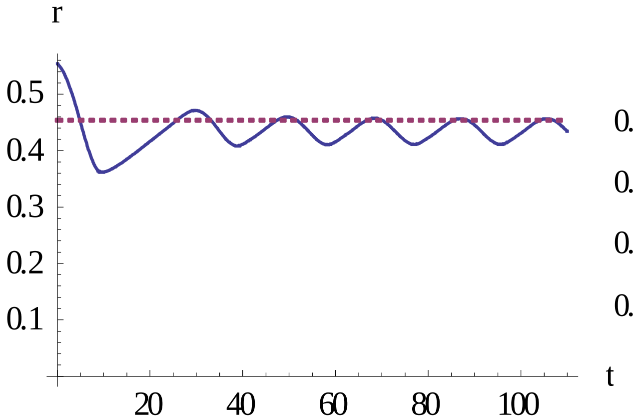


(c) e

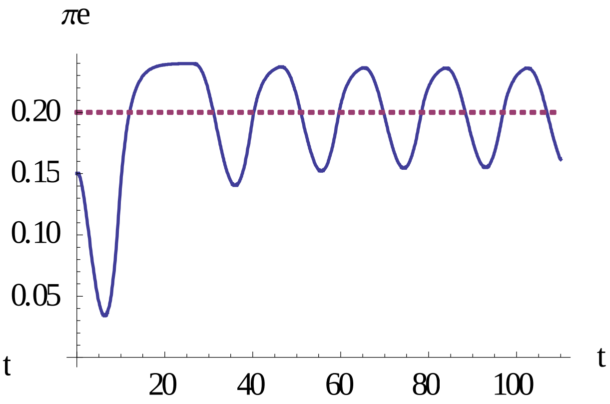


(d) r, g, b

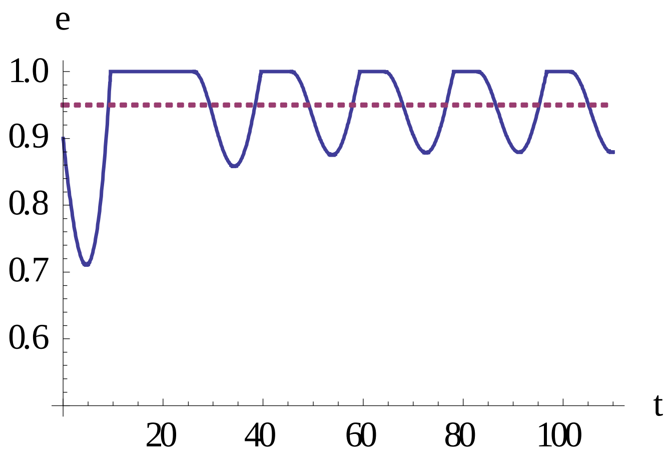
Figure 1. Case A ($\xi = 0.4$, $\theta = 0.3$, $\beta_1 = 0.04$, $\beta_2 = 0.04$)



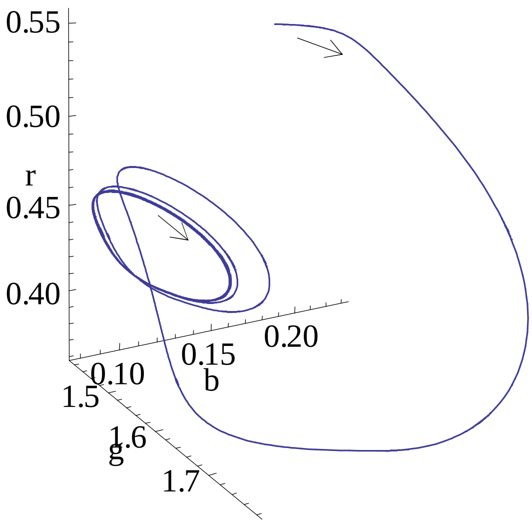
(a) r



(b) π^e

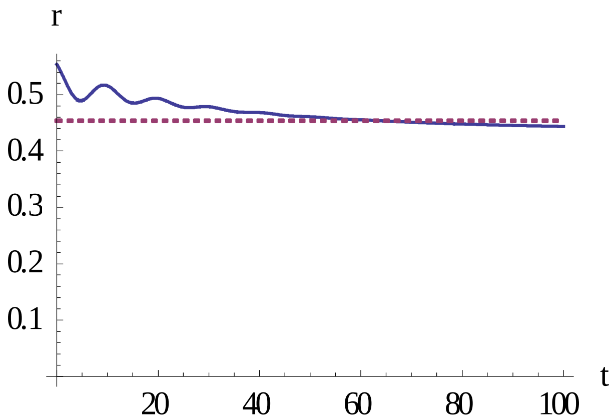


(c) e

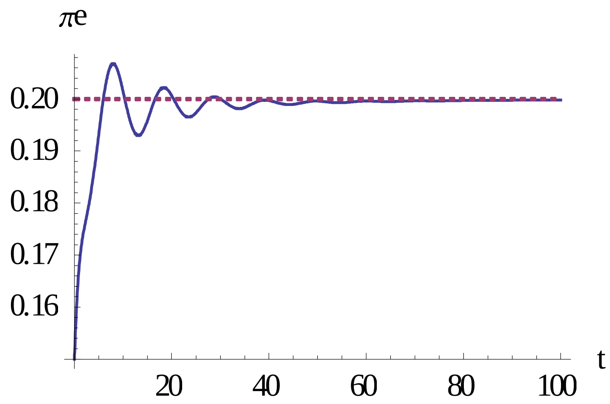


(d) r , g , b

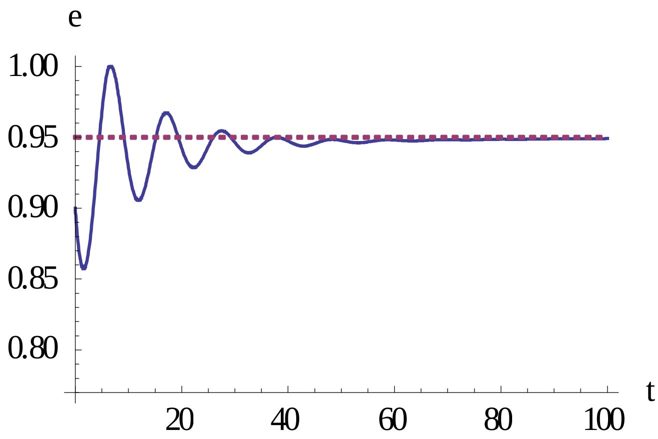
Figure 2. Case B　($\xi = 0.5$,　$\theta = 0.5$,　$\beta_1 = 0.05$,　$\beta_2 = 0.05$)



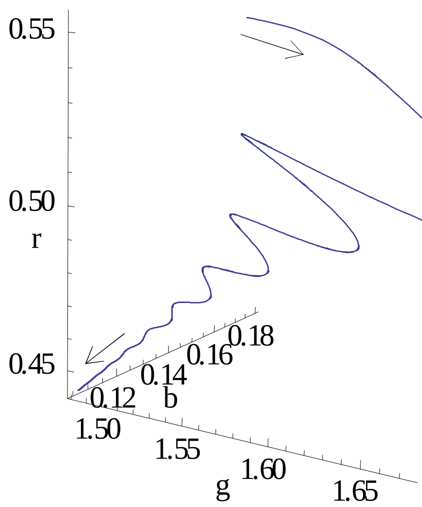
(a) r



(b) π^e



(c) e



(d) r, g, b

Figure 3. Case C ($\xi = 0.8$, $\theta = 0.8$, $\beta_1 = 0.1$, $\beta_2 = 0.1$)

inflation expectation in a forward looking way on the basis of the announced target rate of inflation.[20]

Next, let us try to interpret intuitively why these two propositions are logically correct.

If the government expenditure responds sensitively to the changes of employment, the stabilizing negative feedback mechanism such as

$$e\downarrow\Rightarrow g\uparrow\Rightarrow (c+i+g)\uparrow\Rightarrow y\uparrow\Rightarrow e\uparrow \tag{F1}$$

works. On the other hand, if the government expenditure responds sensitively to the changes of public debt, the destabilizing positive feedback mechanism such that

$$b\uparrow\Rightarrow g\downarrow\Rightarrow y\downarrow, t\downarrow, h\downarrow\Rightarrow b\uparrow \tag{F2}$$

outweighs the stabilizing negative feedback mechanism $b\uparrow\Rightarrow g\downarrow\Rightarrow b\downarrow$ (see equations (3), (5), and (6)). In this case, another destabilizing positive feedback mechanism

$$y\downarrow\Rightarrow t\downarrow\Rightarrow b\uparrow\Rightarrow g\downarrow\Rightarrow (c+i+g)\downarrow\Rightarrow y\downarrow \tag{F3}$$

exists.

If the publics' inflation expectation is formed adaptively, the destabilizing positive feedback mechanism through the effect of the changes of the real rate of interest on the change of the investment expenditure (so called 'Mundell effect') such that

$$e\downarrow\Rightarrow \pi\downarrow\Rightarrow \pi^e\downarrow\Rightarrow (r-\pi^e)\uparrow\Rightarrow i\downarrow\Rightarrow (c+i+g)\downarrow\Rightarrow y\downarrow\Rightarrow e\downarrow \tag{F4}$$

exists, and this destabilizing effect becomes stronger as the adjustment speed of inflation expectation (γ) increases.

On the other hand, if the central bank's inflation targeting is credible, the stabilizing feedback mechanism such that

$$\bar{\pi} > \pi^e \Rightarrow \pi^e\uparrow\Rightarrow (r-\pi^e)\downarrow\Rightarrow i\uparrow\Rightarrow (c+i+g)\uparrow\Rightarrow y\uparrow\Rightarrow e\uparrow \tag{F5}$$

works even if the initial reduction of e induces the decrease of π^e temporarily.

Needless to say, the central bank's active monetary policy (large values of β_1 and β_2) contributes to stabilize the economy through the quick response of the central bank to the changes of macroeconomic environment.

Acknowledgment

During this research, T. Asada was financially supported by the Japan Society for

[20] Curiously enough, this appropriate policy mix is often dismissed by the policy makers especially in Japan. As a result, during the period of the 1990s and the 2000s in Japan, the deflationary depression that accompanied the 'liquidity trap', in which the nominal rate of interest was stuck to its lower bound, was caused (see Krugman 1998 and Asada 2010, 2011).

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Mathematical Appendices

Appendix A : Partial Derivatives

$$F_{41} = (\partial F_4 / \partial g)^* = [\frac{(1-\tau)(1-\delta)}{1-\delta(1-\tau)} - \sigma(\bar{b} + h^*)]/\{1+\psi(r^*)\underset{(+)}{y_b}^*\}. \quad \text{(A1)}$$

$$F_{42} = (\partial F_4 / \partial r)^* = [-\tau y_r^* + \{(1-\tau) - \underset{(-)}{i_{r-\pi^e}}^*\}\bar{b} - \underset{(-)}{i_{r-\pi^e}}^* h^*]/\{1+\psi(r^*)\underset{(+)}{y_b}^*\}. \quad \text{(A2)}$$

$$F_{43} = (\partial F_4 / \partial \pi^e)^* = [-\tau \underset{(+)}{y_{\pi^e}}^* - \bar{b} + \underset{(-)}{i_{r-\pi^e}}^*(\bar{b} + h^*) - \{\bar{\pi} + \frac{\varepsilon_0 + n_s}{1-\varepsilon}\}\psi(r^*)\underset{(+)}{y_{\pi^e}}^*$$

$$-\{\underset{(-)}{\psi'(r^*)}\, y^* + \psi(r^*) y_r^*\}\beta_1 + \psi(r^*)\underset{(+)}{y_{\pi^e}}^*\gamma\xi]/\{1+\psi(r^*)\underset{(+)}{y_b}^*\}. \quad \text{(A3)}$$

$$F_{44} = (\partial F_4 / \partial b)^* = [-\tau \underset{(+)}{y_b}^* + \{r^*(1-\tau) - \bar{\pi} - \frac{\varepsilon_0 + n_s}{1-\varepsilon}\} - \{\bar{\pi} + \frac{\varepsilon_0 + n_s}{1-\varepsilon}\}\psi(r^*)\underset{(+)}{y_b}^*$$

$$+\psi(r^*)\underset{(+)}{y_g}^*\alpha(1-\theta)]/\{1+\psi(r^*)\underset{(+)}{y_b}^*\}. \quad \text{(A4)}$$

$$F_{45} = (\partial F_4 / \partial e)^* = [-\kappa - \{\underset{(-)}{\psi'(r^*)} + \psi(r^*) y_r^*\}(\kappa\beta_1 + \beta_2)$$

$$+\psi(r^*)\{-\underset{(+)}{y_{\pi^e}}^*\gamma(1-\xi)\kappa + \underset{(+)}{y_g}^*\alpha\theta\}]/\{1+\psi(r^*)\underset{(+)}{y_b}^*\}. \quad \text{(A5)}$$

$$F_{51} = (\partial F_5 / \partial g)^* = \bar{e}[\underset{(+)}{y_b}^* F_{41} / y^* + (1-\varepsilon)\sigma]. \quad \text{(A6)}$$

$$F_{52} = (\partial F_5 / \partial r)^* = \bar{e}[\underset{(+)}{y_b}^* F_{42} / y^* + (1-\varepsilon)\underset{(-)}{i_{r-\pi^e}}^*]. \quad \text{(A7)}$$

$$F_{53} = (\partial F_5 / \partial \pi^e)^* = (\bar{e} / y^*)[y_r^* \beta_1 - \underset{(+)}{y_{\pi^e}}^*\gamma\xi + \underset{(+)}{y_b}^* F_{43} - (1-\varepsilon)\underset{(-)}{i_{r-\pi^e}}^*]. \quad \text{(A8)}$$

$$F_{54} = (\partial F_5 / \partial b)^* = (\bar{e} / y^*)[\underset{(+)}{y_b}^* F_{44} - \underset{(+)}{y_g}^*\alpha(1-\theta)]. \quad \text{(A9)}$$

$$F_{55} = (\partial F_5 / \partial e)^* = (\bar{e} / y^*)[y_r^*(\kappa\beta_1 + \beta_2) + \underset{(+)}{y_{\pi^e}}^*\gamma(1-\xi)\kappa + \underset{(+)}{y_b}^* F_{45}$$

$$-\underset{(+)}{y_g}*\alpha\theta]. \tag{A10}$$

The asterisk(*) means that the values are evaluated *at the equilibrium point*.

Appendix B : Proof of Proposition 1.

Suppose that $\theta=\beta_1=\beta_2=\xi=0$. In this case, we have the following expression (see equations (43), (A4), (A5) and (A10)).

$$a_1=[\tau\underset{(+)}{y_b}*-r*(1-\tau)+\{\bar{\pi}+\frac{\varepsilon_0+n_s}{1-\varepsilon}\}\{1+\psi(r*)\underset{(+)}{y_b}*\}-\psi(r*)\underset{(+)}{y_g}*\alpha]/\{1+\psi(r*)\underset{(+)}{y_b}*\}$$

$$+(\bar{e}/y*)[-\underset{(+)}{y_{\pi^e}}*\gamma\kappa+\underset{(+)}{y_b}*\{\kappa+\psi(r*)\underset{(+)}{y_{\pi^e}}*\gamma\kappa\}/\{1+\psi(r*)\underset{(+)}{y_b}*\}]. \tag{B1}$$

It follows from Eq. (B1) that we have $a_1<0$ for all sufficiently large values of $\alpha>0$, which means that one of the necessary conditions for the local stability is violated for all sufficiently large values of $\alpha>0$ if $\theta=\beta_1=\beta_1=\xi=0$. By continuity, this conclusion is qualitatively unaffected even if $0<\theta<1$, $\beta_1>0$, $\beta_2>0$ and $0<\xi<1$, as long as all of them are sufficiently close to 0. □

Appendix C : Proof of Proposition 2.

Assume Assumptions 1 and 2, and suppose that $\xi=1$. In this case, the Jacobian matrix (41) becomes as follows.

$$J=\begin{bmatrix} 0 & 0 & 0 & -\alpha(1-\theta) & -\alpha\theta \\ 0 & 0 & \beta_1 & 0 & \kappa\beta_1+\beta_2 \\ 0 & 0 & -\gamma & 0 & 0 \\ F_{41} & F_{42} & F_{43} & F_{44} & F_{45} \\ F_{51} & F_{52} & F_{53} & F_{54} & F_{55} \end{bmatrix} \tag{C1}$$

Then, the characteristic equation (42) becomes

$$\Gamma(\lambda)=|\lambda I-J|=|\lambda I-J_4|(\lambda+\gamma)=0, \tag{C2}$$

where

$$J_4 = \begin{bmatrix} 0 & 0 & -\alpha(1-\theta) & -\alpha\theta \\ 0 & 0 & 0 & \kappa\beta_1+\beta_2 \\ F_{41} & F_{42} & F_{44}(\theta) & F_{45}(\beta_1,\beta_2,\theta) \\ F_{51} & F_{52} & F_{54}(\theta) & F_{55}(\beta_1,\beta_2,\theta) \end{bmatrix}. \tag{C3}$$

Under Assumptions 1 and 2, we have

$$F_{41} > 0,\ \ F_{42} > 0,\ \ F_{44}(1) < 0,\ \ F_{51} > 0,\ \ F_{52} < 0,\ \ F_{54}(1) < 0. \tag{C4}$$

The characteristic equation (C2) has a negative real root $\lambda_5 = -\gamma$, and other four roots are determined by the following equation.

$$\Gamma_4(\lambda) = |\lambda I - J_4| = \lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4 = 0, \tag{C5}$$

where

$$b_1 = -trace J_4 = -F_{44}(\theta) - F_{55}(\beta_1,\beta_2,\theta), \tag{C6}$$

b_2 = sum of all principal second-order minors of J_4

$$= \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -\alpha(1-\theta) \\ F_{41} & F_{44}(\theta) \end{vmatrix} + \begin{vmatrix} 0 & -\alpha\theta \\ F_{51} & F_{55}(\beta_1,\beta_2,\theta) \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ F_{42} & F_{44}(\theta) \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & \kappa\beta_1+\beta_2 \\ F_{52} & F_{55}(\beta_1,\beta_2,\theta) \end{vmatrix} + \begin{vmatrix} F_{44}(\theta) & F_{45}(\beta_1,\beta_2,\theta) \\ F_{54}(\theta) & F_{55}(\beta_1,\beta_2,\theta) \end{vmatrix}$$

$$= \alpha(1-\theta)\underset{(+)}{F_{41}} + \alpha\theta\underset{(+)}{F_{51}} - (\kappa\beta_1+\beta_2)\underset{(-)}{F_{52}} + F_{44}(\theta)F_{55}(\beta_1,\beta_2,\theta)$$

$$- F_{45}(\beta_1,\beta_2,\theta)F_{54}(\theta), \tag{C7}$$

$b_3 = -($ sum of all principal third-order minors of $J_4)$

$$= -\begin{vmatrix} 0 & 0 & \kappa\beta_1+\beta_2 \\ F_{42} & F_{44}(\theta) & F_{45}(\beta_1,\beta_2,\theta) \\ F_{52} & F_{54}(\theta) & F_{55}(\beta_1,\beta_2,\theta) \end{vmatrix} - \begin{vmatrix} 0 & -\alpha(1-\theta) & -\alpha\theta \\ F_{41} & F_{44}(\theta) & F_{45}(\beta_1,\beta_2,\theta) \\ F_{51} & F_{54}(\theta) & F_{55}(\beta_1,\beta_2,\theta) \end{vmatrix}$$

$$- \begin{vmatrix} 0 & 0 & -\alpha\theta \\ 0 & 0 & F_{45}(\beta_1,\beta_2,\theta) \\ F_{51} & F_{52} & F_{55}(\beta_1,\beta_2,\theta) \end{vmatrix} - \begin{vmatrix} 0 & 0 & -\alpha(1-\theta) \\ 0 & 0 & 0 \\ F_{41} & F_{42} & F_{44}(\theta) \end{vmatrix}$$

$$= -(\kappa\beta_1 + \beta_2)\begin{vmatrix} F_{42} & F_{44}(\theta) \\ F_{52} & F_{54}(\theta) \end{vmatrix} - \alpha(1-\theta)\begin{vmatrix} F_{41} & F_{45}(\beta_1, \beta_2, \theta) \\ F_{51} & F_{55}(\beta_1, \beta_2, \theta) \end{vmatrix} + \alpha\theta\begin{vmatrix} F_{41} & F_{44}(\theta) \\ F_{51} & F_{54}(\theta) \end{vmatrix}$$

$$= (\kappa\beta_1 + \beta_2)\{-F_{42}F_{54}(\theta) + F_{44}(\theta)F_{52}\} + \alpha(1-\theta)A(\beta_1, \beta_2, \theta) + \alpha\theta B(\theta)$$

$$= (\kappa\beta_1 + \beta_2)[(1-\varepsilon)\underset{(-)}{i_{r-\pi^e}} * F_{44}(\theta) - \underset{(+)}{y_g} * \alpha(1-\theta)\underset{(+)}{F_{42}}]$$

$$+ \alpha(1-\theta)A(\beta_1, \beta_2, \theta) + \alpha\theta B(\theta), \tag{C8}$$

$$b_4 = \det J_4 = -\alpha(1-\theta)\begin{vmatrix} 0 & 0 & \kappa\beta_1 + \beta_2 \\ F_{41} & F_{42} & F_{45}(\beta_1, \beta_2, \theta) \\ F_{51} & F_{52} & F_{55}(\beta_1, \beta_2, \beta_3) \end{vmatrix}$$

$$= \alpha(1-\theta)(\kappa\beta_1 + \beta_2)(-F_{41}F_{52} + F_{42}F_{51})$$

$$= \alpha(1-\theta)(\kappa\beta_1 + \beta_2)(1-\varepsilon)\hat{e}[-\underset{(-)}{i_{r-\pi^e}} * \underset{(+)}{F_{41}} + \sigma \underset{(+)}{F_{42}}] > 0 \quad \text{if } 0 < \theta < 1. \tag{C9}$$

It is well known that the Routh-Hurwitz conditions for stable roots of the characteristic equation (C5) are given by the following set of inequalities (see mathematical appendices of Asada, Chiarella, Flaschel and Franke 2003, 2010).

$$b_j > 0 \text{ for all } j \in \{1,2,3,4\}, \quad \Phi = b_1 b_2 b_3 - b_1^2 b_4 - b_3^2 > 0 \tag{C10}$$

Suppose that the parameter θ is close to 1 and either of the parameters β_1 or β_2 is sufficiently large. Then, it is easy to see that the inequalities $b_1 > 0$, $b_2 > 0$, and $b_3 > 0$ are satisfied under Assumptions 1 and 2. On the other hand, we always have $b_4 > 0$ as long as $0 < \theta < 1$.

Furthermore, we have

$$\lim_{\theta \to 1} \Phi = \lim_{\theta \to 1} (b_1 b_2 - b_3) b_3 \tag{C11}$$

because of the fact that $\lim_{\theta \to 1} b_4 = 0$. Next, suppose that the parameter β_2 is fixed at any positive value. In this case, we can easily see that $b_1 b_2 - b_3$ becomes a quadratic function of β_1 and the coefficient of β_1^2 becomes positive. Furthermore, in this case b_3 becomes a linear increasing function of β_1. It follows from the above considerations that we obtain

$$\lim_{\theta \to 1} \Phi > 0 \tag{C12}$$

for all sufficiently large values of $\beta_1 > 0$. It is worth noting that we can easily interchange the roles of β_1 and β_2 in the above reasoning.

Therefore, all of the Routh-Hurwitz conditions for local stability are satisfied under the conditions that (1) the parameter θ is less than 1, but it is close to 1, (2) parameter α is fixed at any positive value, (3) either of the parameters β_1 or β_2 is sufficiently large, and (4) $\xi = 1$. By continuity, however, the local stability result also applies in case of $0 < \xi < 1$, as long as ξ is sufficiently close to 1. □

Appendix D : Calculation of the Coefficient a_5 in the General Case

From equations (41) and (45) we obtain the following relationship.

$$a_5 = -\det J = \alpha\gamma \begin{vmatrix} 0 & 0 & 0 & (1-\theta) & \theta \\ 0 & 0 & \beta_1 & 0 & \kappa\beta_1 + \beta_2 \\ 0 & 0 & -\xi & 0 & (1-\xi)\kappa \\ F_{41} & F_{42} & F_{43} & F_{44} & F_{45} \\ F_{51} & F_{52} & F_{53} & F_{54} & F_{55} \end{vmatrix}$$

$$= \alpha\gamma[-\xi \begin{vmatrix} 0 & 0 & (1-\theta) & \theta \\ 0 & 0 & 0 & \kappa\beta_1 + \beta_2 \\ F_{41} & F_{42} & F_{44} & F_{45} \\ F_{51} & F_{52} & F_{54} & F_{55} \end{vmatrix} + (1-\xi)\kappa \begin{vmatrix} 0 & 0 & 0 & (1-\theta) \\ 0 & 0 & \beta_1 & 0 \\ F_{41} & F_{42} & F_{43} & F_{44} \\ F_{51} & F_{52} & F_{53} & F_{54} \end{vmatrix}]$$

$$= \alpha\gamma[-\xi(\kappa\beta_1 + \beta_2) \begin{vmatrix} 0 & 0 & (1-\theta) \\ F_{41} & F_{42} & F_{44} \\ F_{51} & F_{52} & F_{54} \end{vmatrix} - (1-\xi)\kappa\beta_1 \begin{vmatrix} 0 & 0 & (1-\theta) \\ F_{41} & F_{42} & F_{44} \\ F_{51} & F_{52} & F_{54} \end{vmatrix}]$$

$$= -\alpha\gamma(1-\theta)\{\xi(\kappa\beta_1 + \beta_2) + (1-\xi)\kappa\beta_1\} \begin{vmatrix} F_{41} & F_{42} \\ F_{51} & F_{52} \end{vmatrix}$$

$$= \alpha\gamma(1-\theta)\{\xi(\kappa\beta_1 + \beta_2) + (1-\xi)\kappa\beta_1\}(-F_{41}F_{52} + F_{42}F_{51})$$

$$= \alpha(1-\theta)(1-\varepsilon)\{\xi(\kappa\beta_1 + \beta_2) + (1-\xi)\kappa\beta_1\}[-\underset{(-)}{i_{r-\pi^e}} * F_{41} + \sigma F_{42}]. \tag{D1}$$

Under Assumptions 1 and 2, we have $F_{41} > 0$ and $F_{42} > 0$ (cf. equations (A1) and (A2)). In this case, we always have $a_5 > 0$ as long as $0 \leqq \theta < 1$, $\beta_1 > 0$, $\beta_2 > 0$, and $0 \leqq$

$\xi \leqq 1$.

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